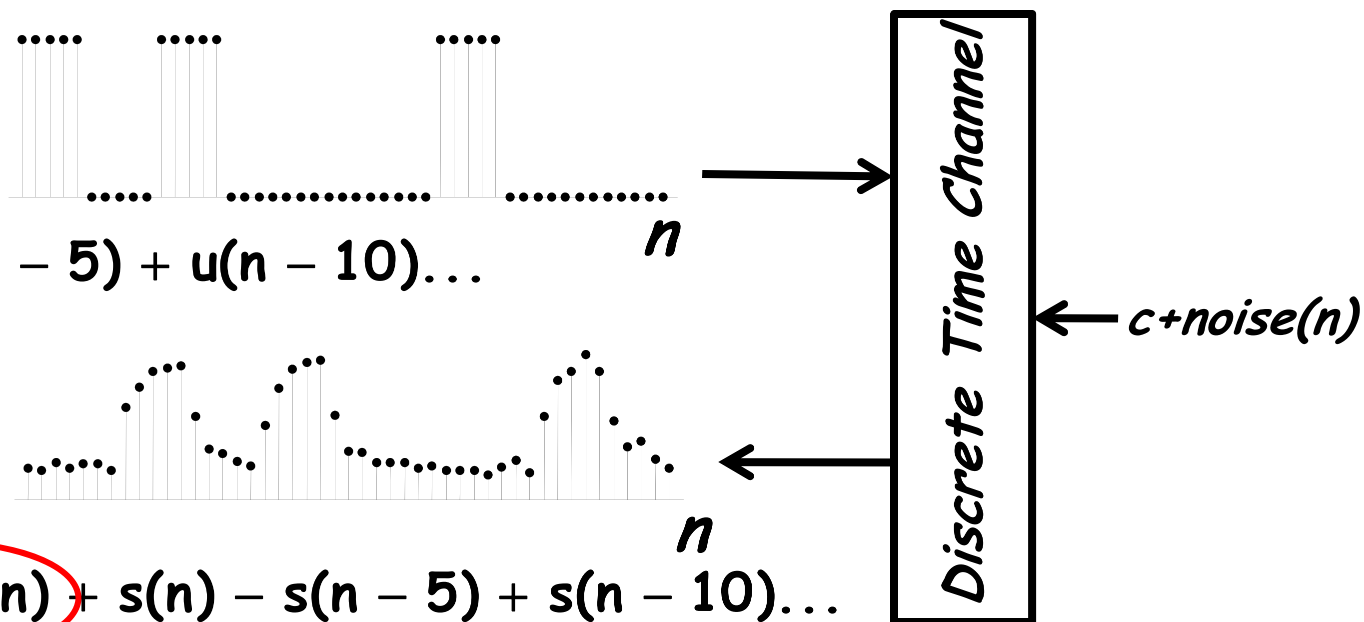
ELEC1200: A System View of Communications: from Signals to Packets Lecture 8

- **Review**
  - Response of a Channel
- **Motivation**
  - Noise: what is it?
  - The effect of noise on a communication channel
- **Analyzing Bit Errors: Part 1**
  - Binary Communication Channel
  - Probabilistic Analysis
  - Predicting BER

Response of a communication channel

- We assume that the output of a communication channel is the sum of two parts:
  - The response to the input, which can be computed from the step response according to the LTI assumption
  - "Extra" things such as offset (c) and noise that are introduced by the channel itself, and do not depend upon the input.

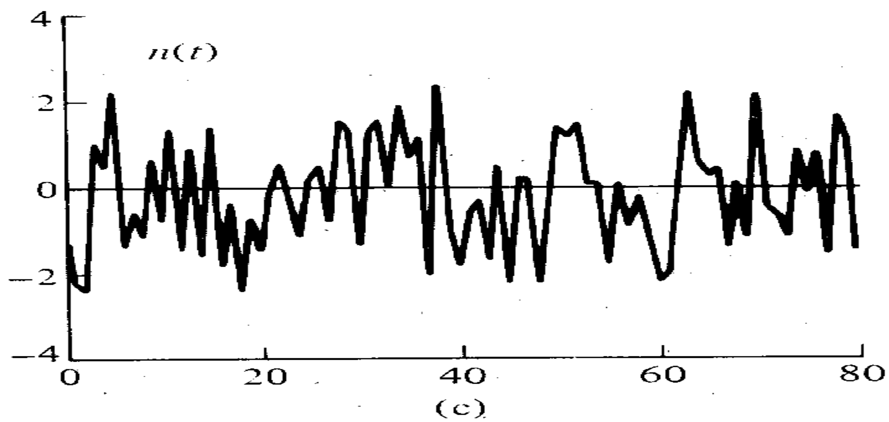


$x(n) = u(n) - u(n-5) + u(n-10)\ldots$

$y(n) = c + noise(n) + s(n) - s(n-5) + s(n-10)\ldots$

Noise

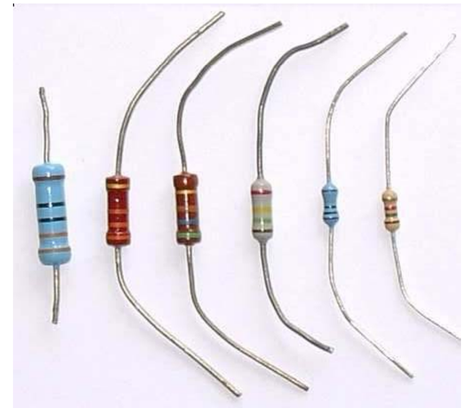
- Noise is one of the most critical and fundamental concepts in communications systems
- Without noise this course would not exist!
- Noise occurs everywhere and a typical noise signal may look like

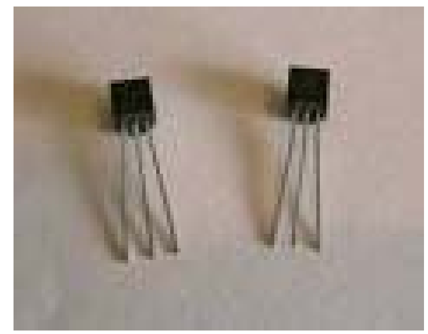


- It is essentially a "random signal"

Where does noise come from?

- Noise occurs naturally in nature and the most common type is thermal noise
- Resisters, devices and the atmosphere are all sources of thermal noise
- Thermal noise is simply due to the ambient heat causing electrons to move and vibrate and create random voltages and emissions
- Noise arises internally in systems as well as externally from such things as the atmosphere



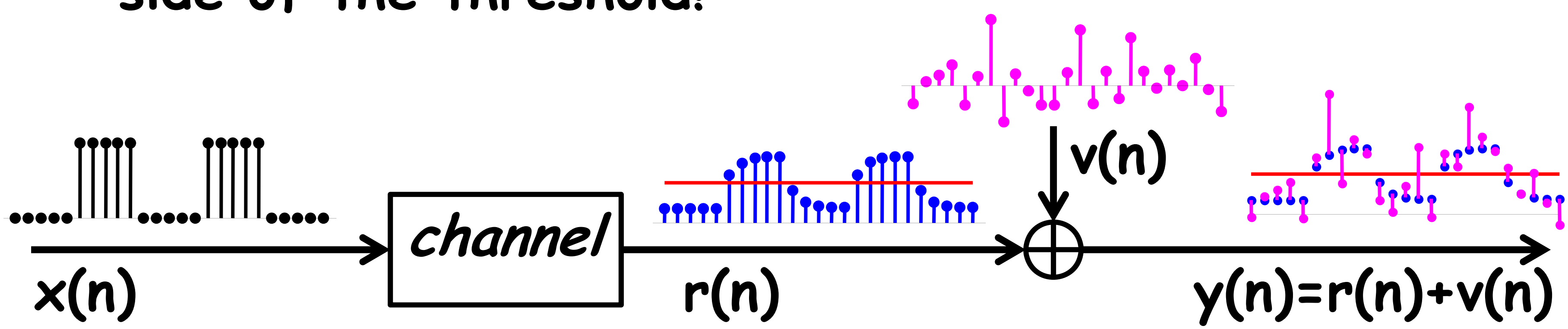


Why is noise so critical?

- If there was no noise, I would be able to talk very, very, very, very quietly and you would still understand me
- The amount of noise determines the minimum signal you can understand.
- It also determines the minimum signal that can be decoded by radios and receivers.
- We like to use small signals to save energy.
- If the desired signal falls below the noise level, bit errors increase significantly.

Additive Noise

- Definitions:
  - x(n): channel input
  - r(n): channel output without noise
  - v(n): noise
  - y(n): received signal
- Additive noise moves the received signal away from the channel output without noise.
- If the noise is large enough and in the right direction, the output sample will be on the wrong side of the threshold!



Simplifying Assumptions for BER Analysis

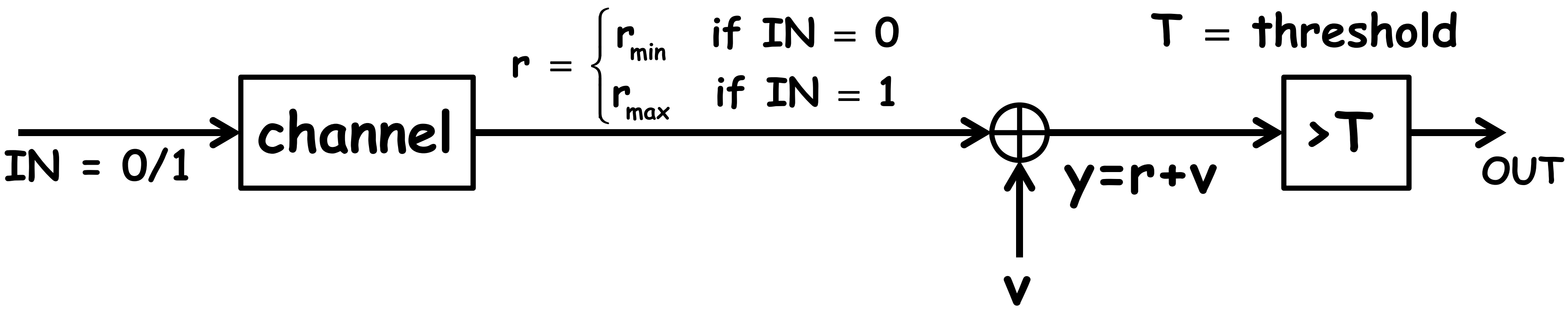
- Perfect synchronization
  - We know exactly where to sample the output to decode each bit.
- Single sample decoding
  - We decode each bit by comparing one output sample with a threshold
- No ISI
  - The channel response depends only on the current bit, and not on past bits.
- Additive "White" Gaussian Noise (AWGN)
  - White: the noise varies fast enough that its value at different samples are unrelated to each other.
  - Gaussian: to be defined next time

Simplified Model

- Under these assumptions, we only need to consider one sample per bit and can analyze each bit in isolation (independently) of the other bits.

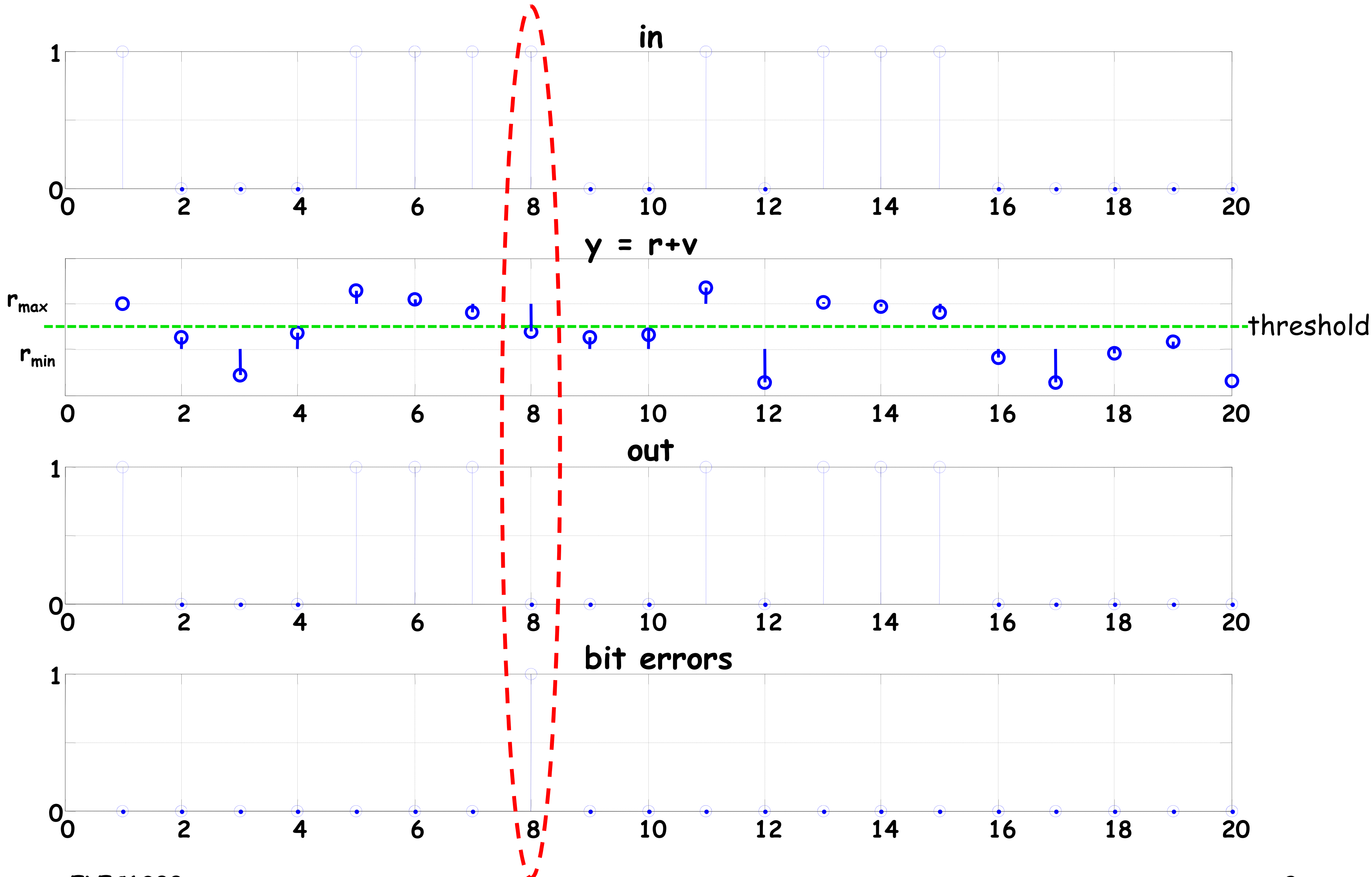
$$OUT = \begin{cases} 0 & \text{if } y < T \\ 1 & \text{if } y \geq T \end{cases}$$

T = threshold



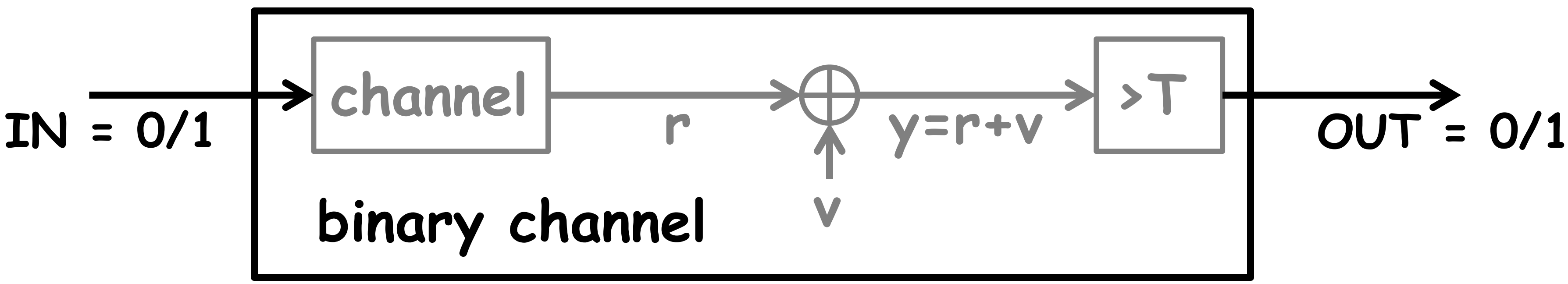
- How can we predict the bit error rate for this model?

Noise leads to bit errors



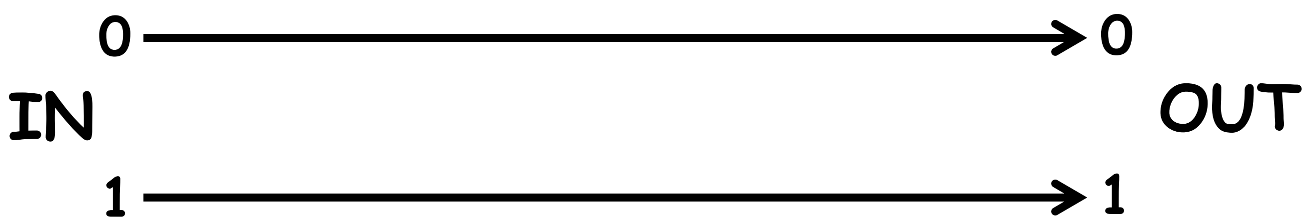
Binary Channel Model

- To start with, we will further simplify the model by ignoring details about the noise and received signal levels r_{min}/r_{max}, but look only at the input and output bits
- Binary channel: both input and output have possible two values, 0 or 1 ("bi" = two).

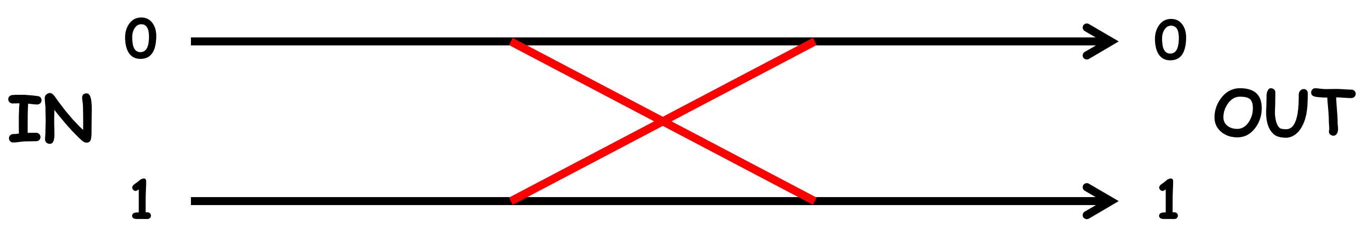


Binary Channel Behavior

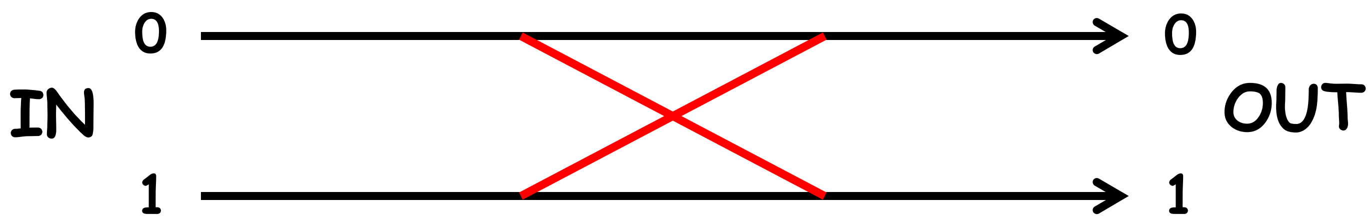
- Ideally, (when IN=0, OUT=0) and (when IN=1, OUT=1). In this case, the BER = 0.



- Unfortunately, due to noise, sometimes (IN=0 but OUT=1) or (IN=1 but OUT=0). In this case, BER > 0.



Probabilistic Analysis



- The BER depends upon
  - "How often" IN = 0, but OUT = 1
  - "How often" IN = 1, but OUT = 0
  - "How often" IN = 0.
  - "How often" IN = 1.

P[IN = 0] = 0.5

- We quantify this notion of "how often" using probability theory.
- Intuitively, the probability of something happening (i.e. IN=0) is the percentage of time that thing happens. For example,

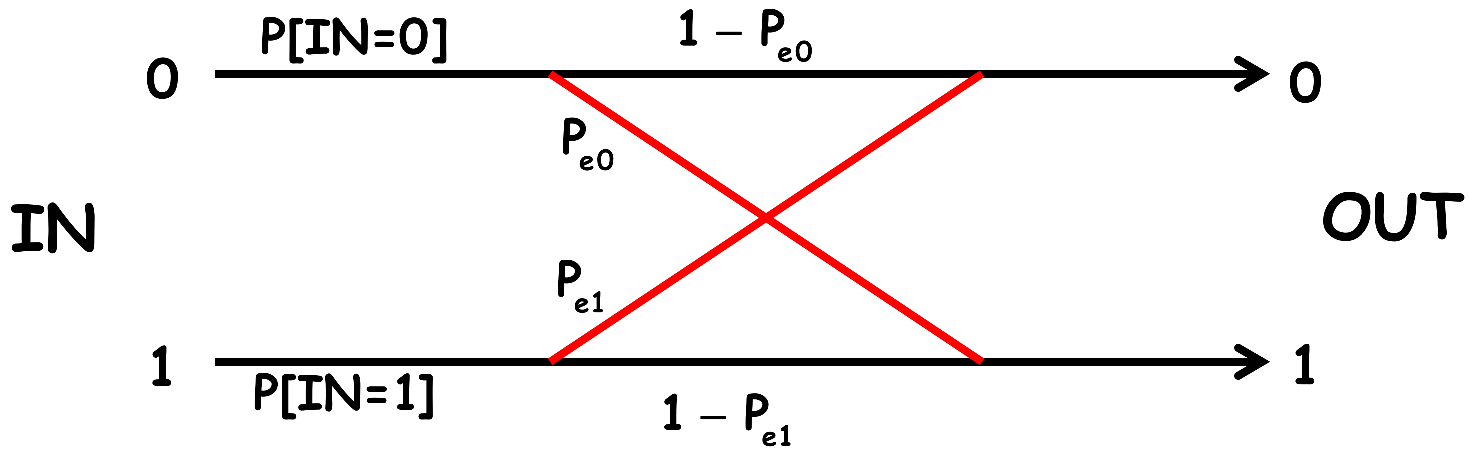
$$P\left[IN = 0\right] = 0.5 \text{ implies that the input bits are zero half the time}$$

- Since there are only two possibilities,

$$P\left[IN = 0\right] + P\left[IN = 1\right] = 1$$

$$P\left[IN = 1\right] = 1 - P\left[IN = 0\right] = 0.5$$

Modeling the Binary Chanel



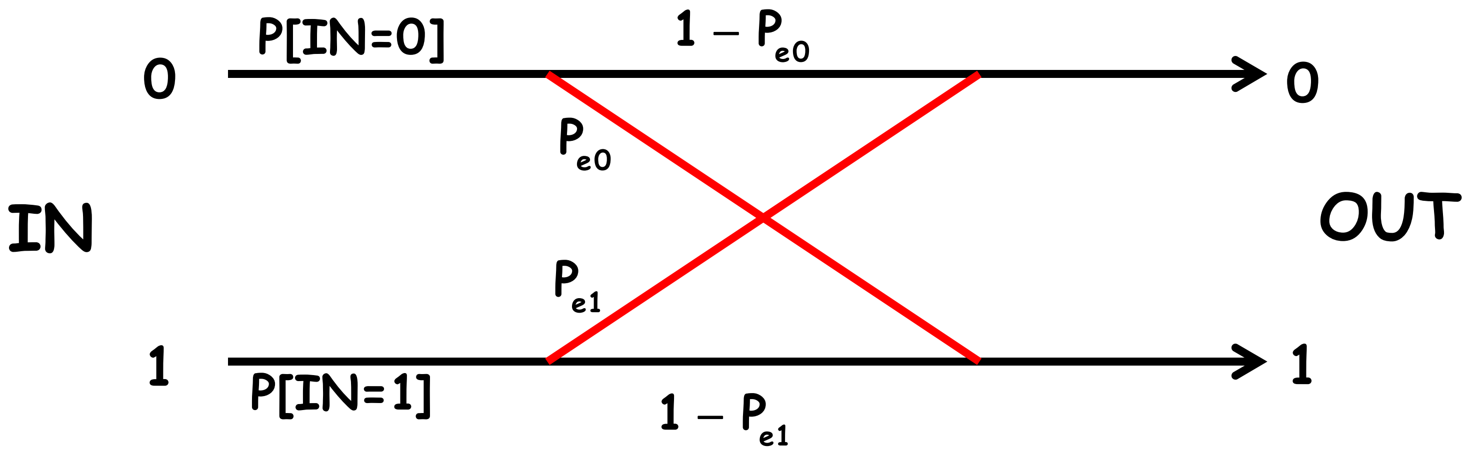
P[IN=0] = probability (% of the time) that IN=0

P[IN=1] = probability (% of the time) that IN=1

P_{e0} = probability (% of the time) there is an error when IN=0
= probability (% of the time) that OUT=1 when IN=0

P_{e1} = probability (% of the time) there is an error when IN=1
= probability (% of the time) that OUT=0 when IN=1

Computing the BER



- The BER is the probability of error, P_e

$$\text{BER} = P_e = \underbrace{P_{e0} \cdot P[\text{IN}=0]}_{\text{probability that OUT=1 and IN=0}} + \underbrace{P_{e1} \cdot P[\text{IN}=1]}_{\text{probability that OUT=0 and IN=1}}$$

probability that OUT=1 and IN=0

probability that OUT=0 and IN=1

Two types of errors!

Example

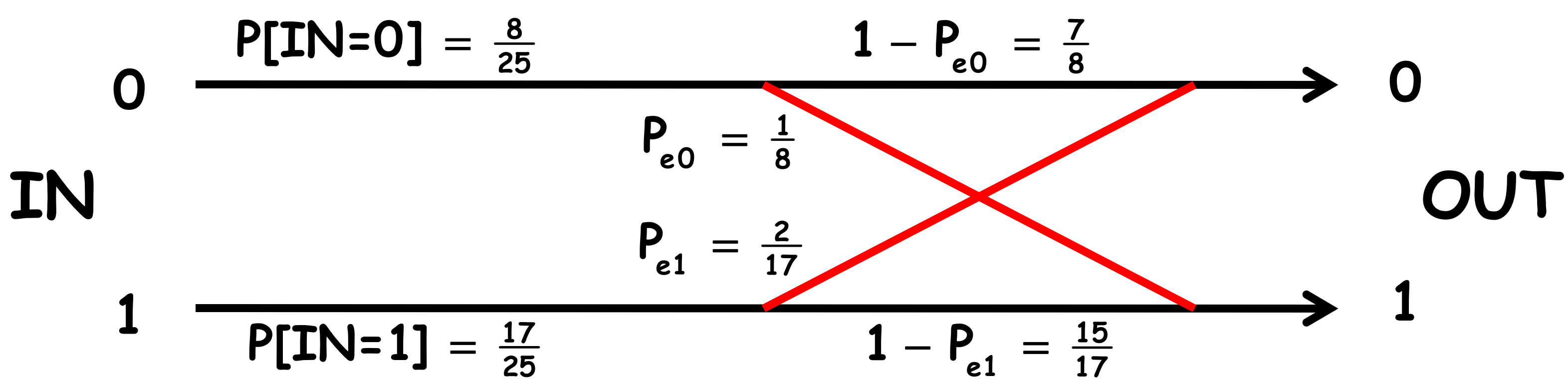
Input/Output Bit Streams

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
IN	1	1	0	1	1	0	0	1	1	1	0	1	1	0	1	0	0	1	1	1	1	0	1	1	1
OUT	1	1	0	1	1	0	0	1	1	1	1	1	1	0	1	0	0	1	1	0	1	0	0	1	1

By definition:

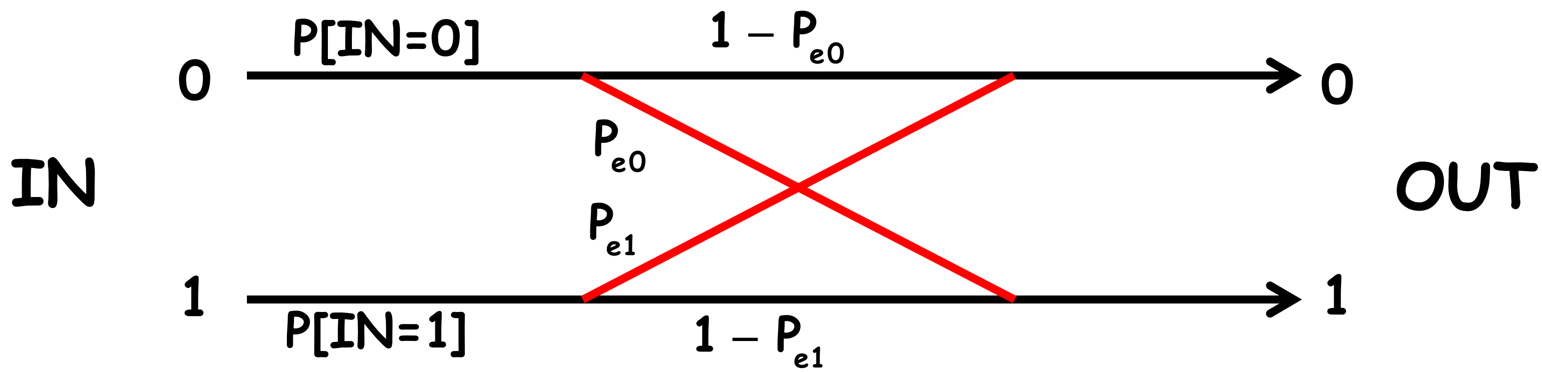
$$BER \approx \frac{\text{\# of errors}}{\text{\# of bit pairs}} = \frac{3}{25} = 12\%$$

Using our formula:



$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1] = \frac{1}{8} \cdot \frac{8}{25} + \frac{2}{17} \cdot \frac{17}{25} = \frac{3}{25}$$

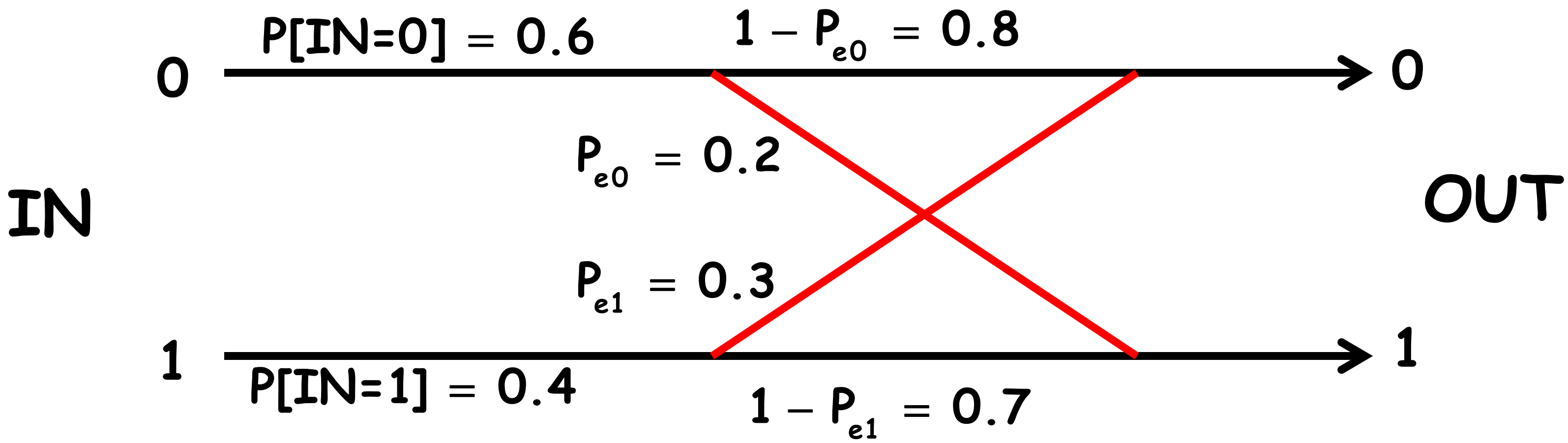
Intuition



$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]$$

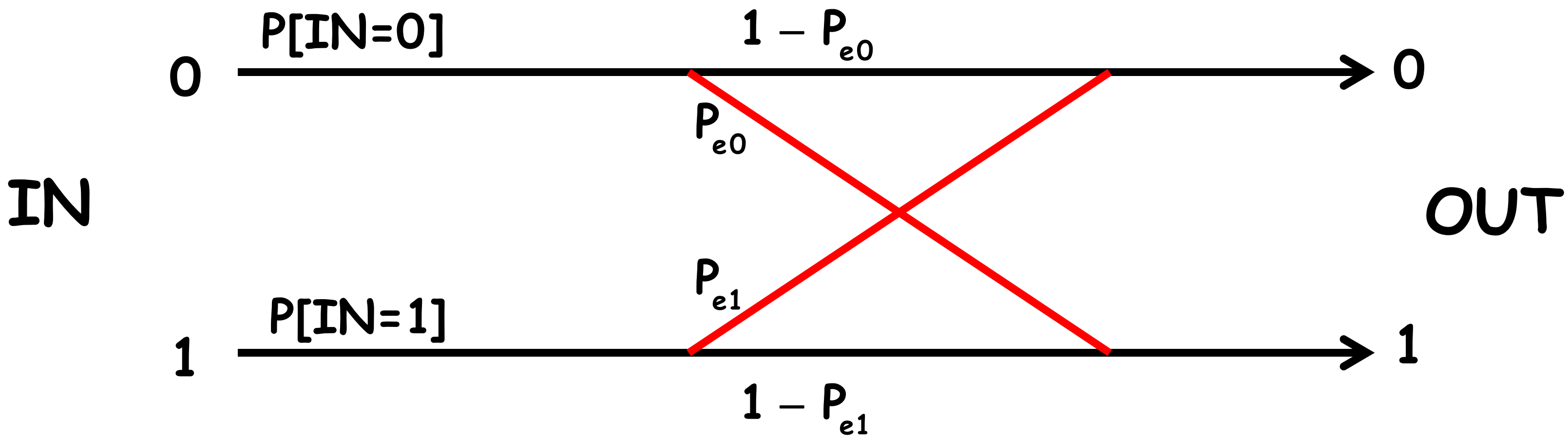
- Since P[IN=0]+P[IN=1]=1,
  - The BER is a weighted average of P_{e0} and P_{e1}
  - The BER is between P_{e0} and P_{e1}
  - If IN=0 is more likely, the BER is closer to P_{e0}
  - If IN=1 is more likely, the BER is closer to P_{e1}
  - If IN=0 and 1 are equally likely, BER = $\frac{1}{2}(P_{e0} + P_{e1})$
  - If $P_{e0} = P_{e1}$, BER = $P_{e0} = P_{e1}$.

Example BER Calculation



What is the BER for the Binary Channel above?

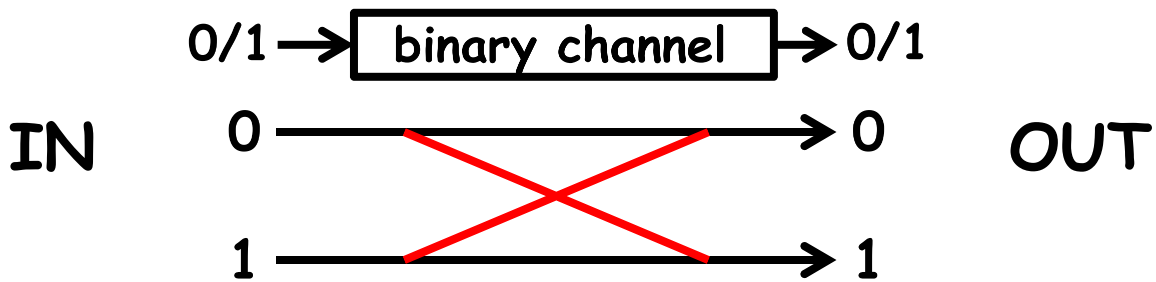
What we need to know to predict BER



- In order to predict the BER, we need to know
  - P[IN=0] (we can find P[IN=1] = 1 - P[IN=0])
  - P_{e0}
  - P_{e1}
- Usually, the transmitter determines P[IN=0]
  - e.g. P[IN=0] = P[IN=1] = 0.5
- P_{e0} and P_{e1} depend on
  - the transmit levels (r_{min},r_{max})
  - the "size" of the noise

Summary

- Noise is one of the critical and fundamental concepts in communications.
- Without noise, there would no difficulty in communication!
- We started our analysis by considering only input/output bits using a simple binary channel model.



- We use probability to get a formula for BER

$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]$$

- Usually, the transmitter controls P[IN=0] and P[IN=1]
  - e.g. P[IN=0] = P[IN=1] = 0.5
- Next lecture: How to determine P_{e0} and P_{e1}?

Connection to Future Classes

- For future reference only!
- ELEC2600
  - The idea that the probability of an event is the % of the time that event happens is known as the "relative frequency interpretation" of probability
  - P_{e0} and P_{e1} are known as "conditional probabilities" because they depend on certain conditions (IN=0 or IN=1)
  - The formula for the BER is an application of the "total probability theorem," which find the total probability of error (the BER) by combining the probability of errors under specific conditions (IN=0 or IN=1).