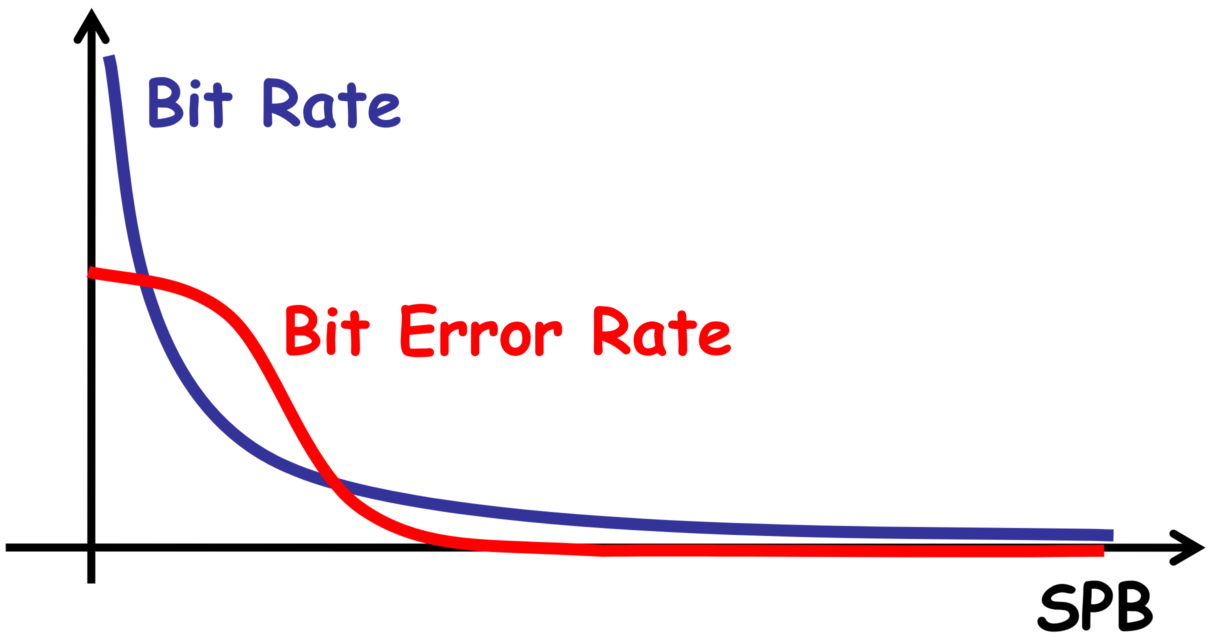
ELEC1200: A System View of Communications: from Signals to Packets
Lecture 5

- Motivation
  - Bit Rate vs. Bit Error Rate (BER)
- Review
  - LTI Channel (superposition)
  - LTI Channel Response to an isolated bit
- LTI channel response to random bit sequences
- Intersymbol interference
- Eye diagrams
- Saturating Ramp Step Response

Trade-off between bit rate and BER

- Bit rate – the number of bits that can be transmitted per second.
  - We want this to be as high as possible
- Bit error rate (BER) – the fraction of bits that are wrongly decoded by the receiver
  - We want this to be low. Usually we place a constraint (maximum limit) on the average BER.
  - BER decreases as the SPB increases
  - BER increases if the channel is noisier or if we use less power to transmit a signal (more on this in later labs).

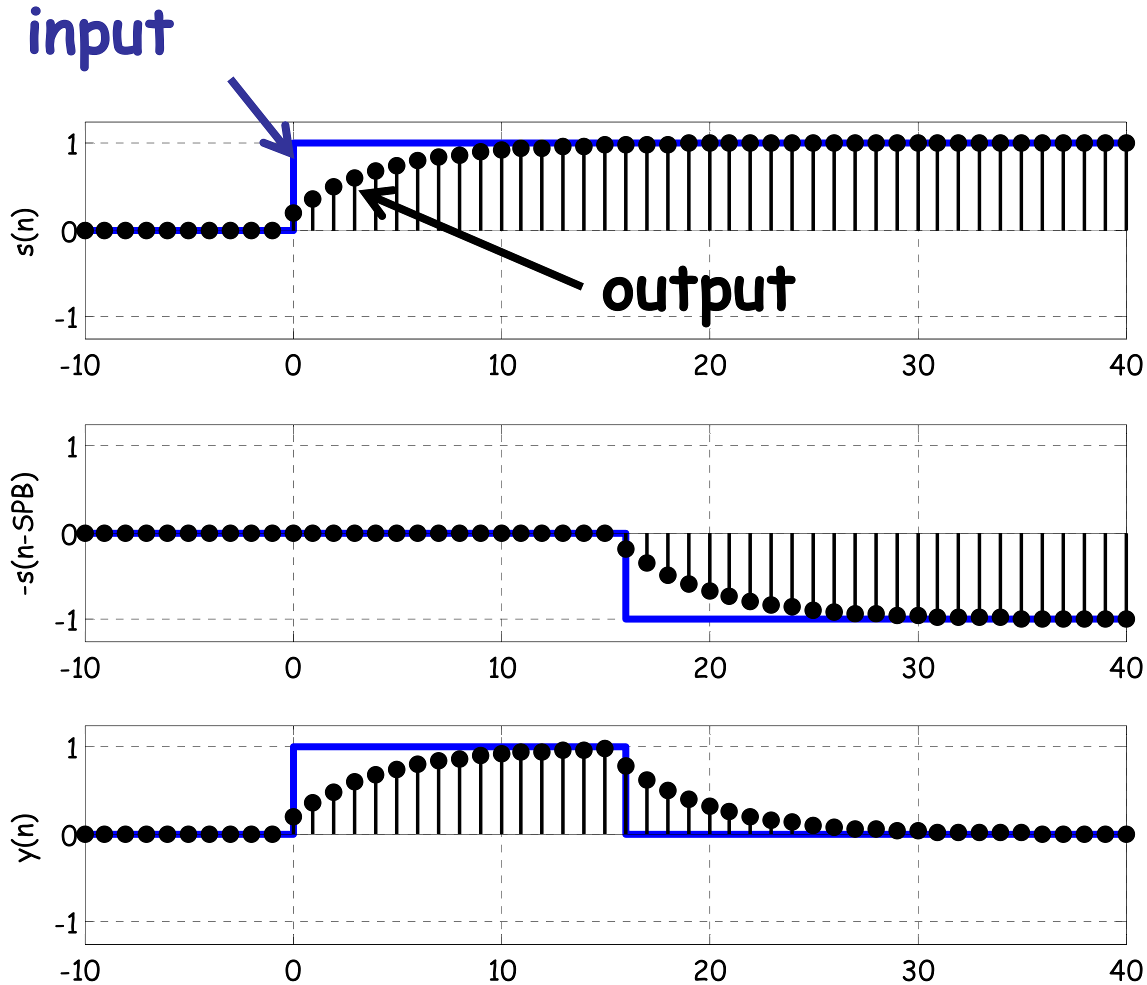


Review: Superposition in an LTI channel

$$x(n) = u(n) - u(n-16)$$

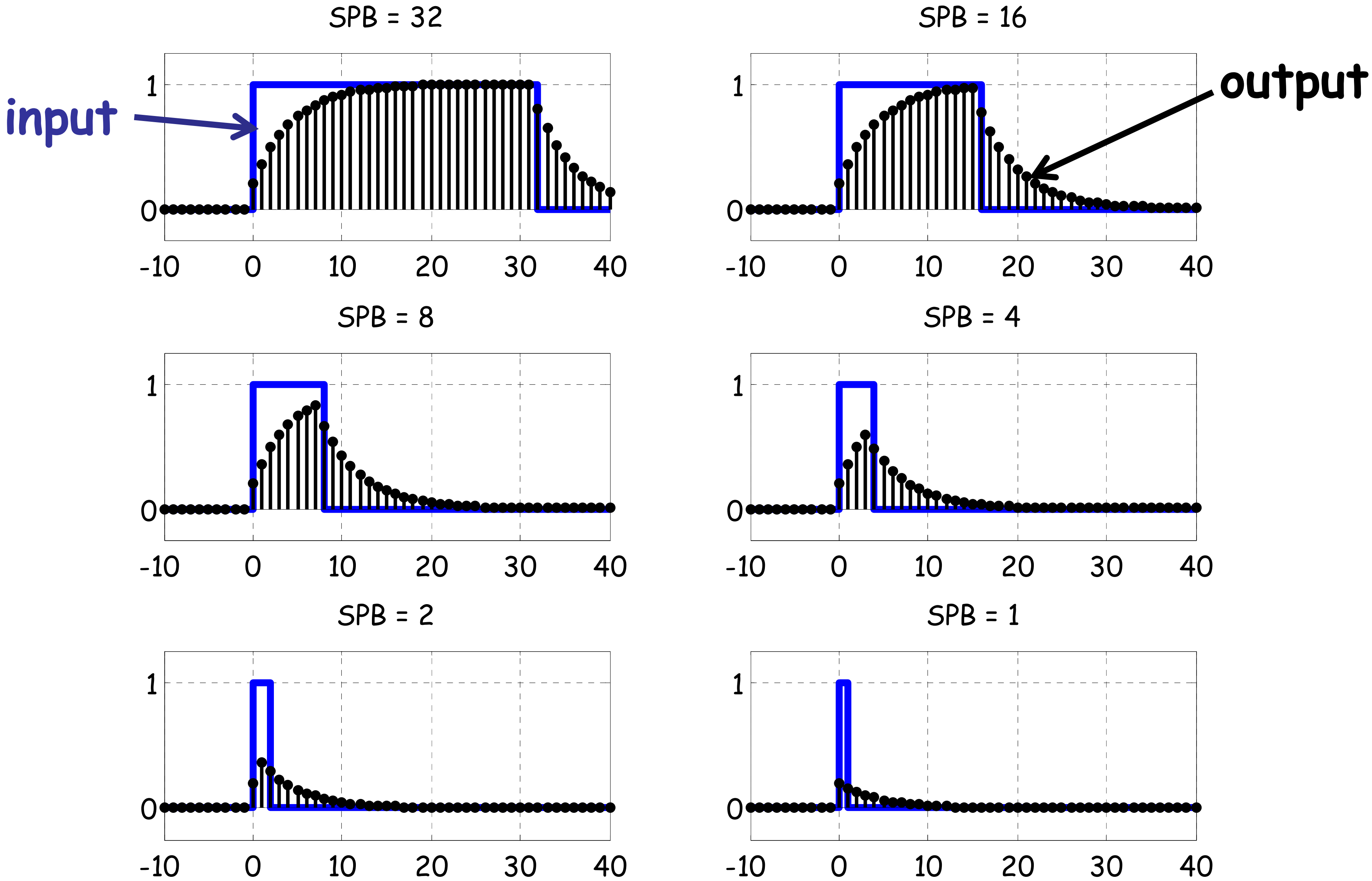
channel | channel | channel

$$y(n) = s(n) - s(n-16)$$



s(n) = Exponential approach, a = 0.8, k = 1, D = 0

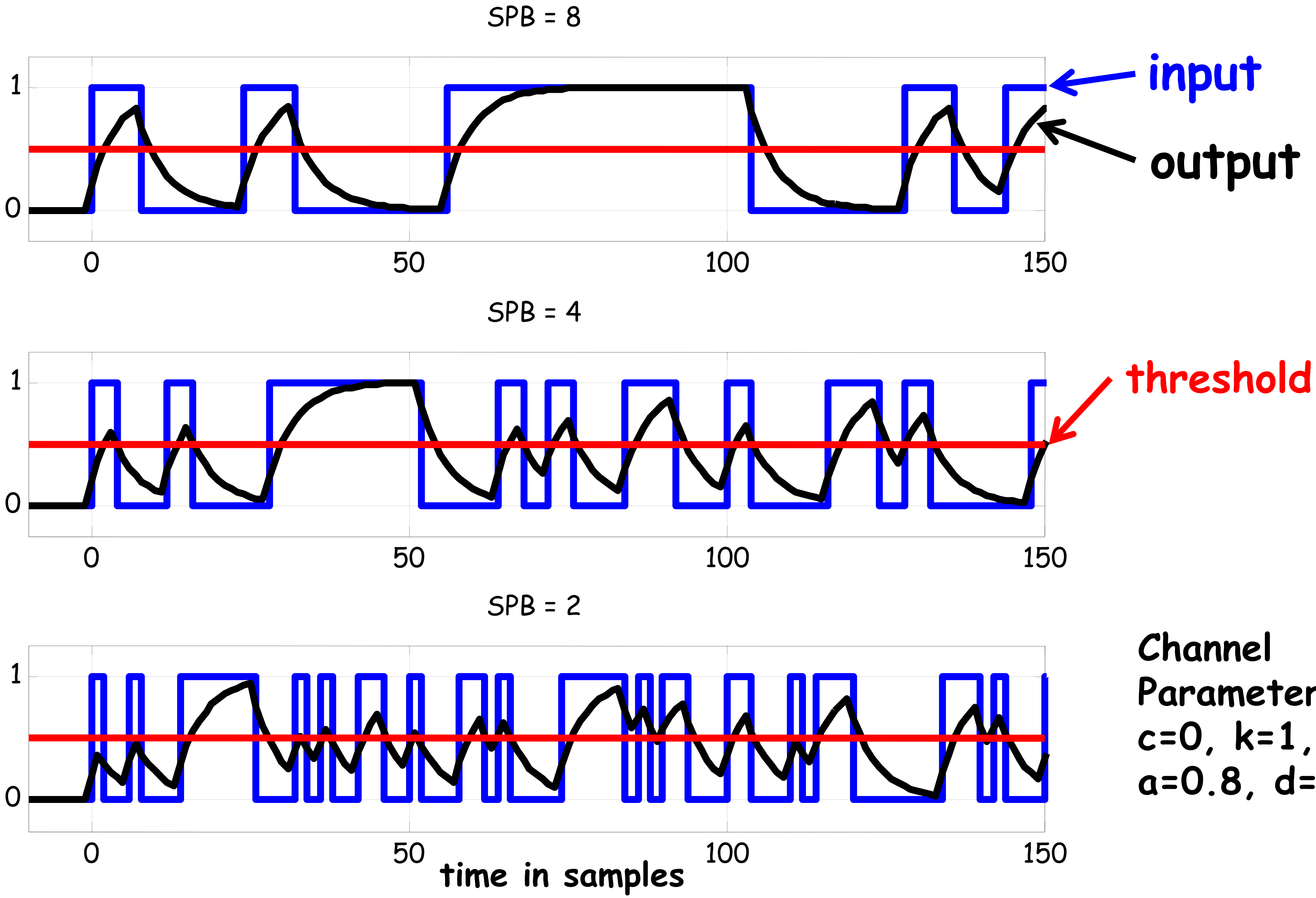
Single Bit Responses (varying SPB)



Effect of SPB

- For a single isolated bit pulse of width SPB, as SPB decreases:
  - The time the response has to rise up to its maximum value decreases.
  - The maximum amplitude decreases
- Intuitively, if we keep the threshold the same (halfway between the minimum value), what should be a one is eventually detected as a zero.
- Reasons for BER increase with smaller SPB:
  - Bit errors due to not crossing threshold

Random Bit Sequence at different SPB



Channel Parameters:
c=0, k=1,
a=0.8, d=0

Observations

- As the SPB decreases or the channel becomes slower,
  - The input and output waveforms become less similar
  - Even if we know where to sample, there are more and more bit errors
    - The response to “one” bits sometimes does not exceed (become greater than) threshold, i.e. “one” bits would be detected as “zero” bits
    - The response to “zero” bits sometimes does not fall below threshold, i.e. “zero” bits would be detected as “one” bits
  - Sometimes the response to the start bit does not exceed threshold, which would lead to synchronization errors (not knowing when to sample)

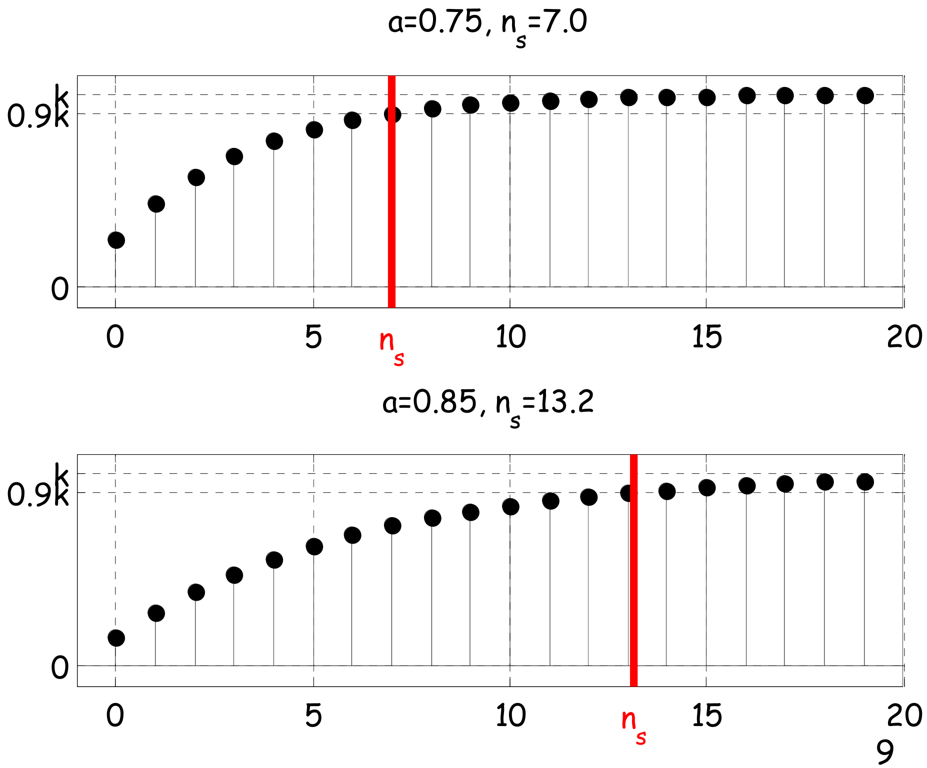
Intersymbol Interference (ISI)

- The response to a "zero" or "one" bit depends upon what bits were transmitted before it, because of the time it takes for the channel to respond to a transition.
- We refer to this as <u>intersymbol interference</u>.
  - Bits = "symbols"
- The smaller the bit time (SPB) in comparison with the time it takes for the channel to respond to a transition, the *greater* the ISI.
  - More past symbols interfere with the current symbol
  - This leads to a larger variety of responses we can observe when a "zero" or "one" bit is transmitted

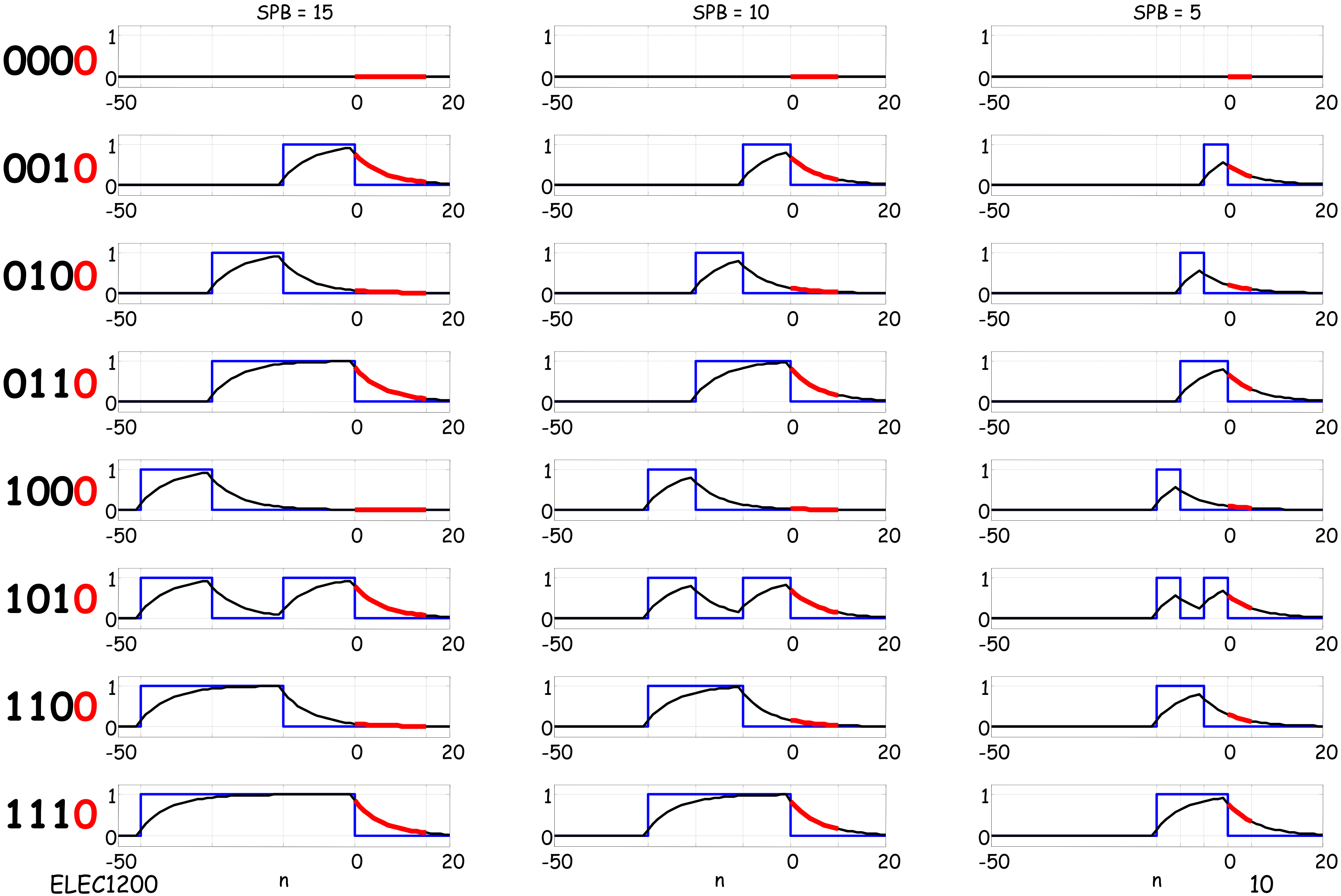
Settling time

- Unless a=0, the exponential response takes an infinite amount of time to reach its final value of k.
- The **settling time** tells us how long it takes for the response to get close to (within 90% of) k.
- To find the settling time, find n such that s(n) = 0.9k, where $s(n) = k(1-a^{n+1})u(n)$ is the exponential step response.

$$
\begin{aligned}
0.9k &= k(1 - a^{n_s+1})u(n) \\
0.9 &= 1 - a^{n_s+1} \\
a^{n_s+1} &= 0.1 \\
(n_s + 1)\ln a &= \ln 0.1 \\
n_s &= \frac{\ln 0.1}{\ln a} - 1
\end{aligned}
$$



Response to zero bit (a=0.85, n_s=13)

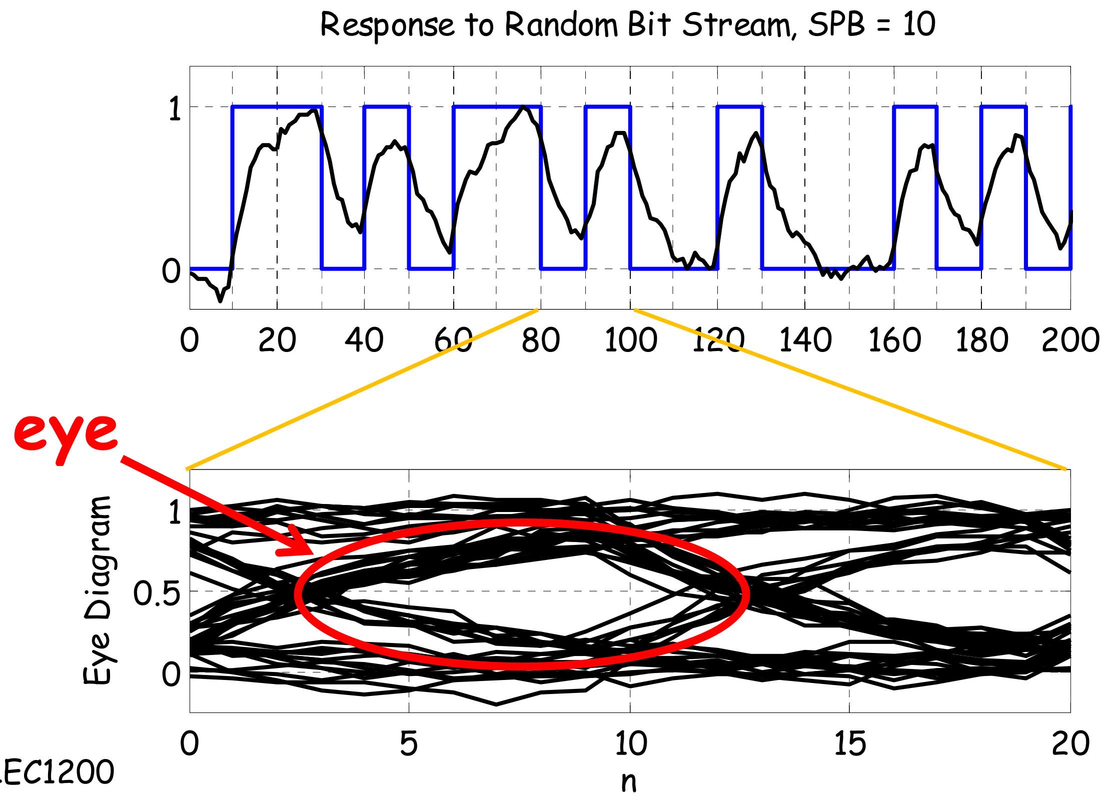


Observations

- The shorter the bit time (SPB), the more variety we observe in the responses to a "0" bit. Roughly speaking,
  - SPB = 15, ~2 different responses
  - SPB = 10, ~4 different responses
  - SPB = 5, ~8 different responses
- The variety depends on the number of previous bits that influence the current bit. Since $n_s = 13$,
  - SPB = 15, the last bit mainly determines the response
  - SPB = 10, the last 2 bits mainly determine the response
  - SPB = 5, the last 3 bits mainly determine the response
- By symmetry, we also observe a similar variety of responses to a "1" bit.

Eye Diagrams

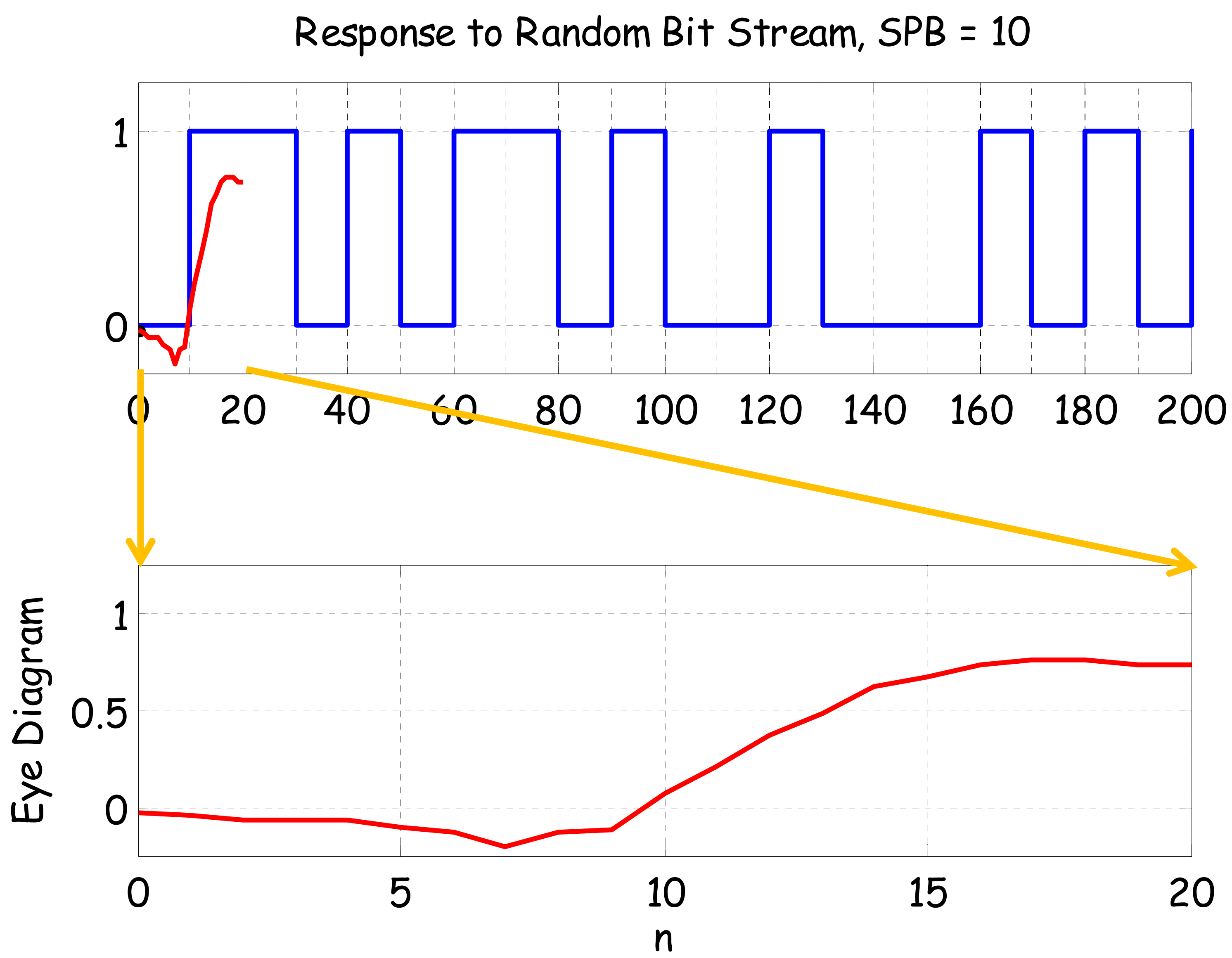
- Eye diagrams summarize the effect of inter symbol interference by showing all responses to “zeros” and “ones” simultaneously.
- We generate an eye diagram by overlaying plots of 2*SPB+1 sample long windows from the response to a random bit stream. Each window is displaced by 2*SPB samples



Exponential Channel
$a=0.85$, $n_s=13$
(noise added)

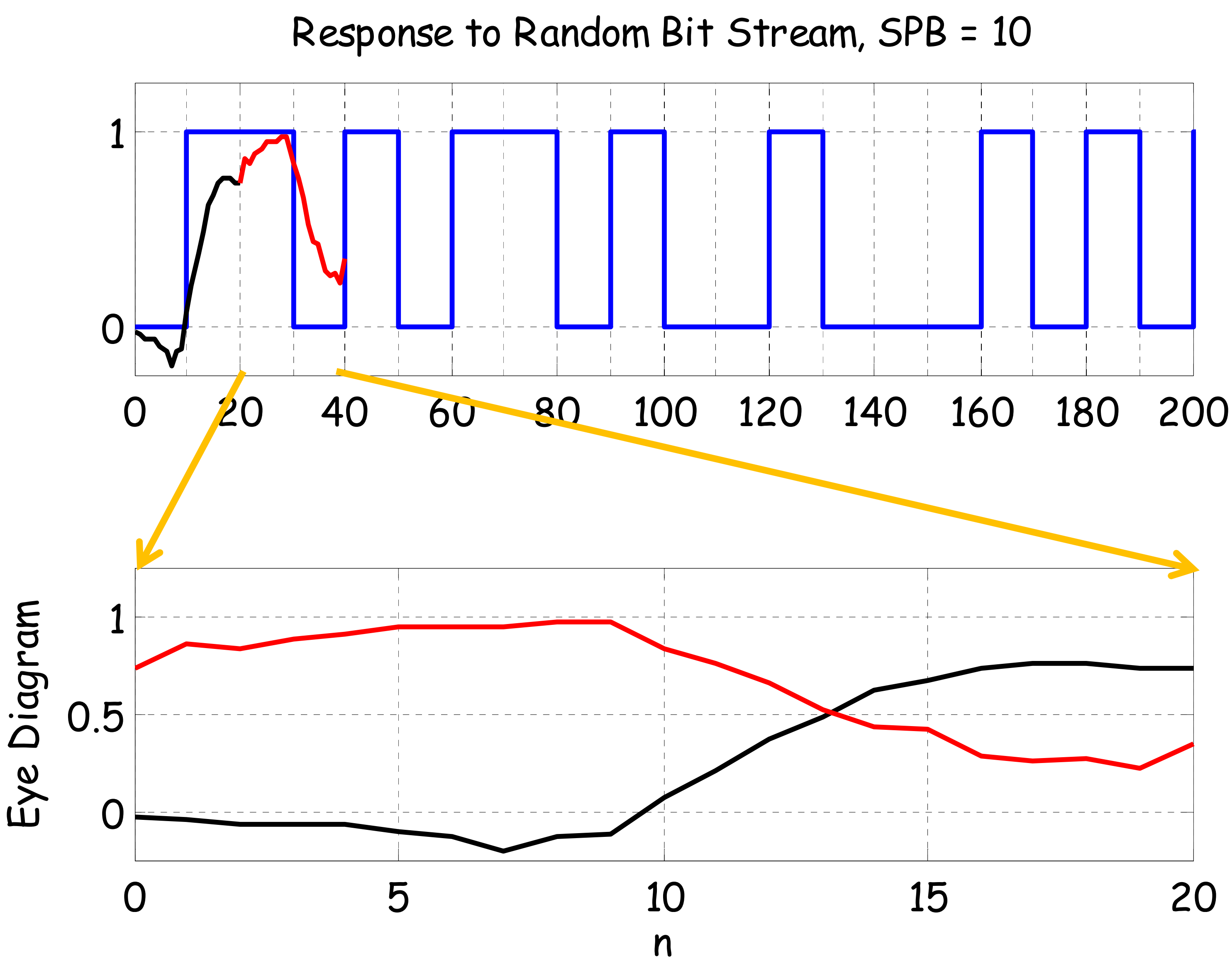
SPB = 10

Construction of the Eye Diagram 1



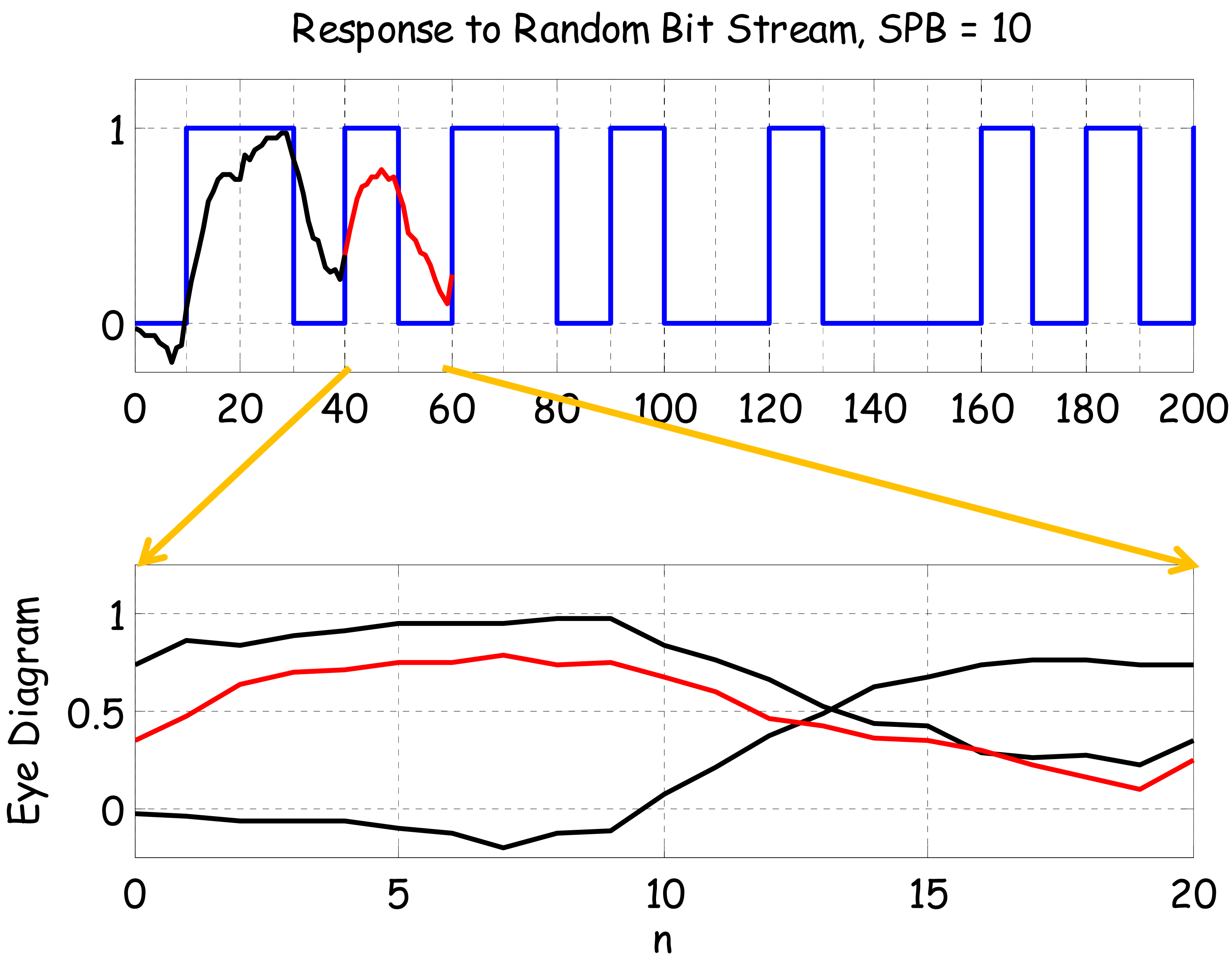
Channel: Exponential Step, a = 0.85, noise added

Construction of the Eye Diagram 2



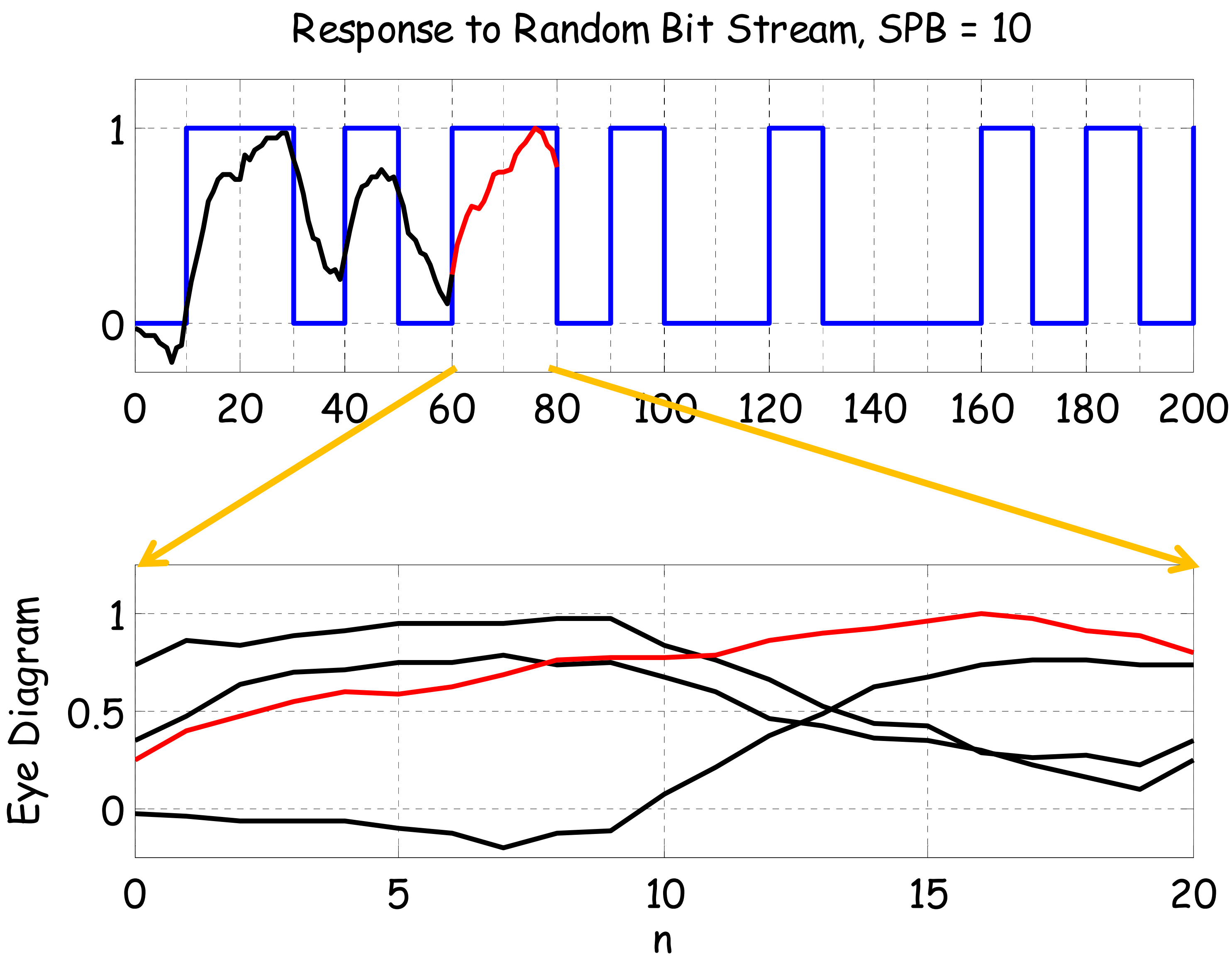
Channel: Exponential Step, a = 0.85, noise added

Construction of the Eye Diagram 3



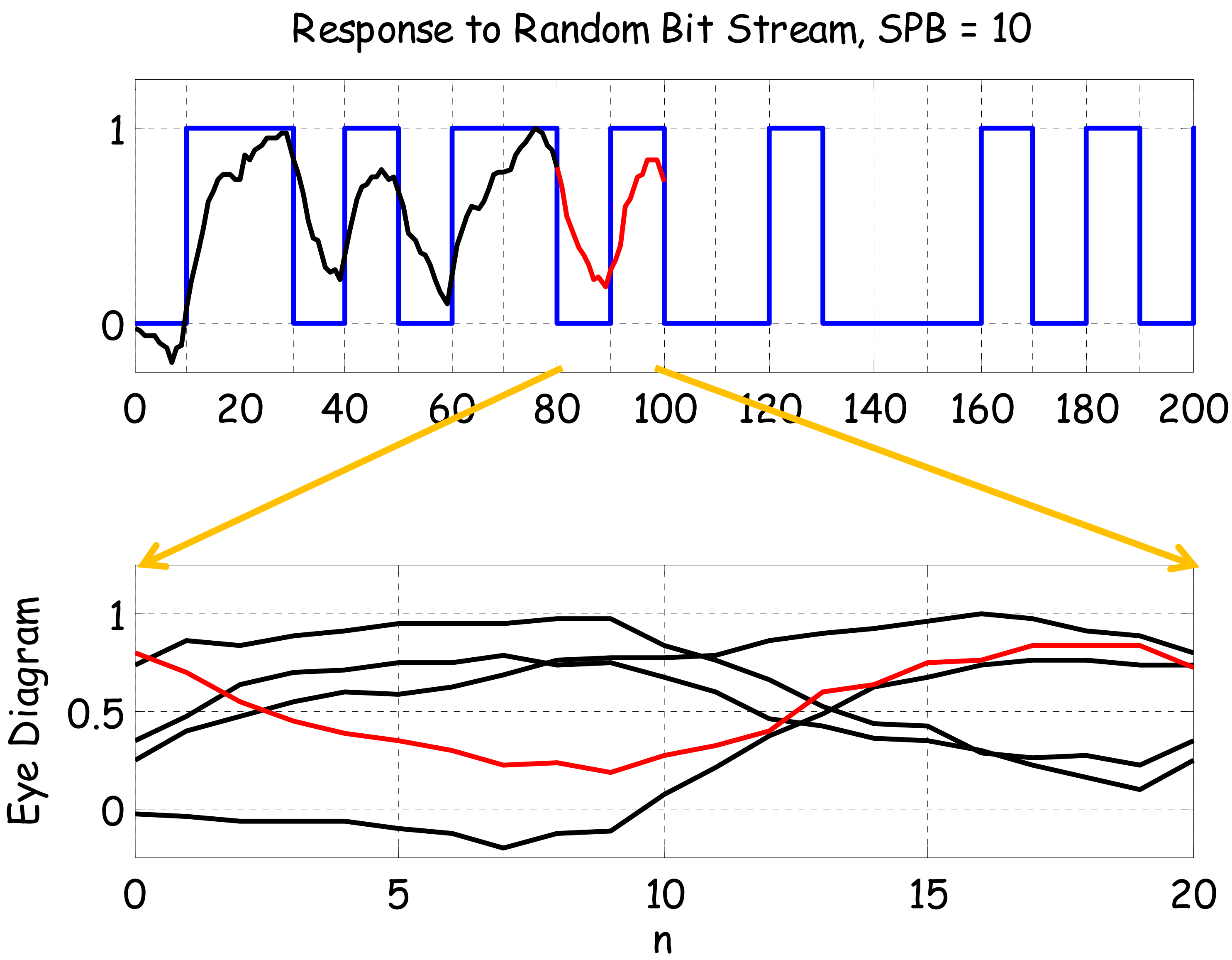
Channel: Exponential Step, a = 0.85, noise added

Construction of the Eye Diagram 4



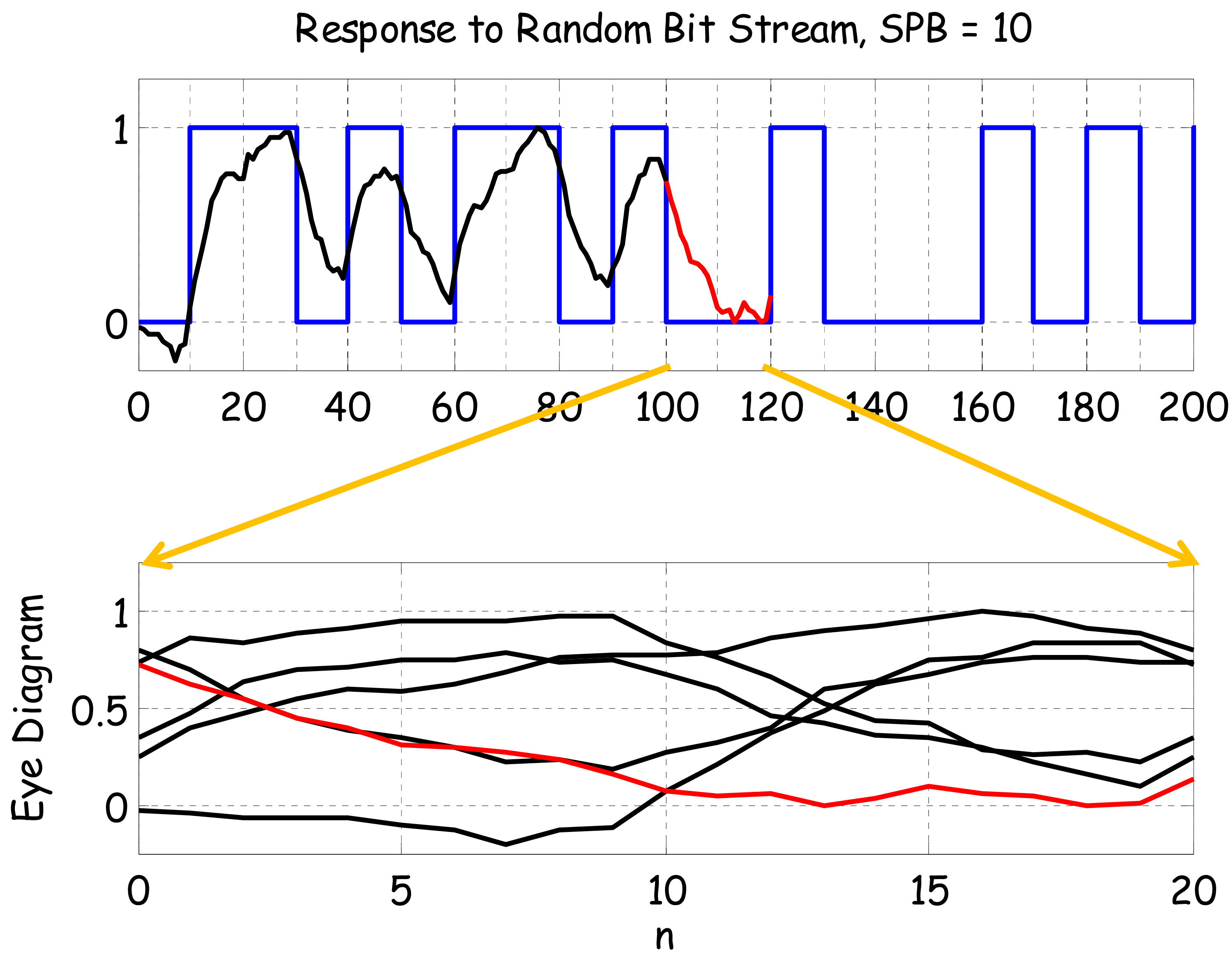
Channel: Exponential Step, a = 0.85, noise added

Construction of the Eye Diagram 5



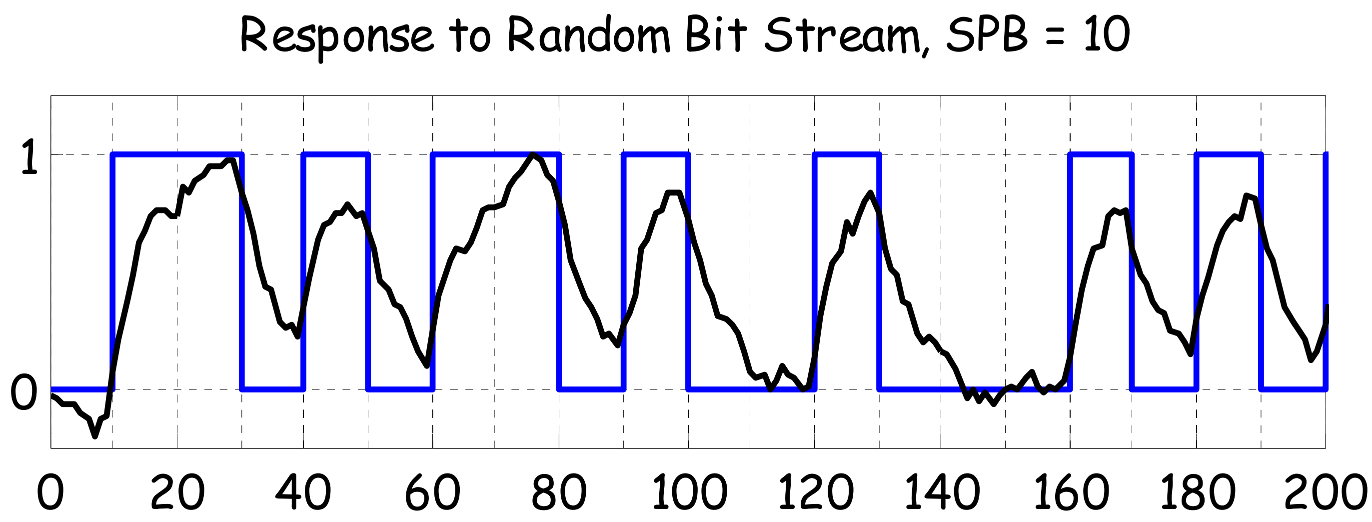
Channel: Exponential Step, a = 0.85, noise added

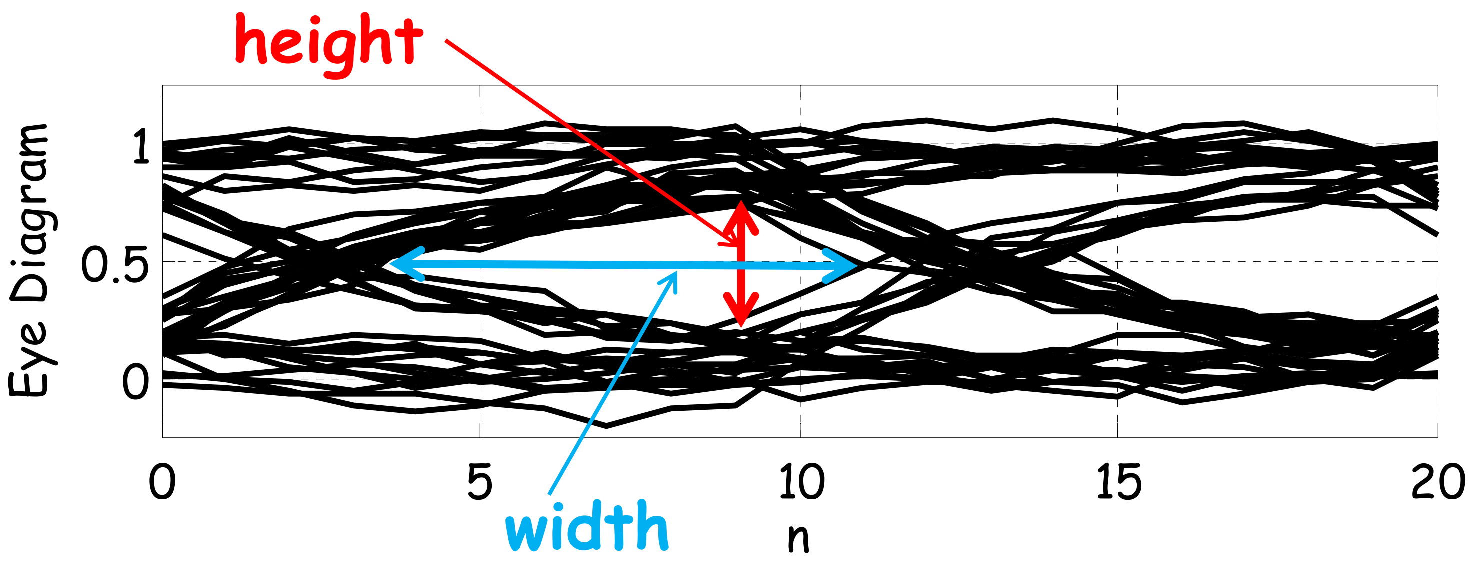
Construction of the Eye Diagram 6



Channel: Exponential Step, a = 0.85, noise added

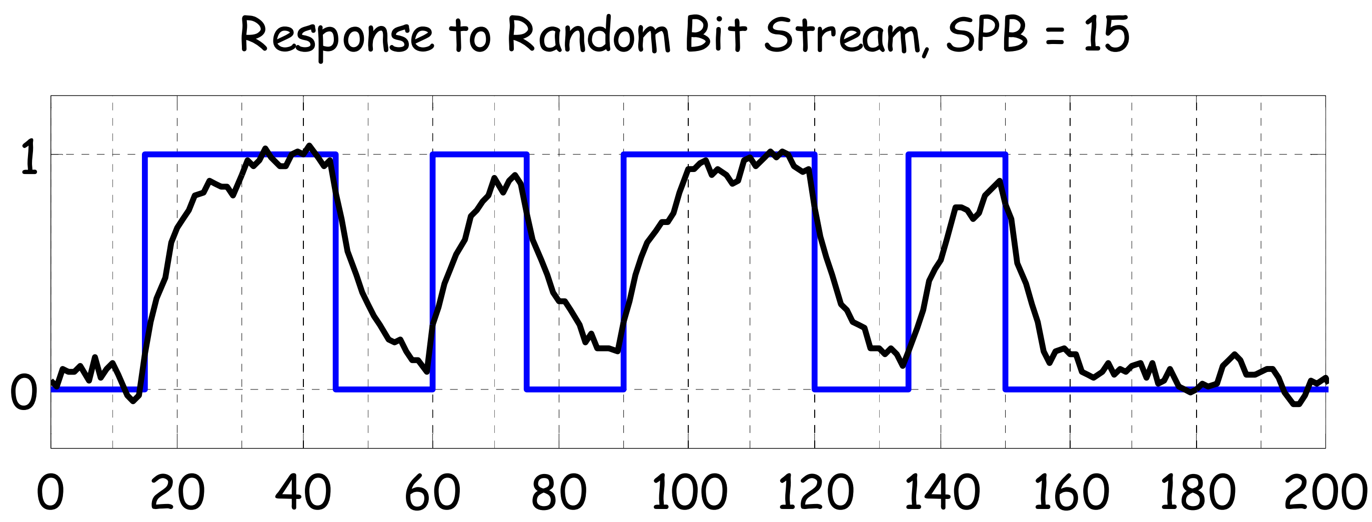
Final Eye Diagram

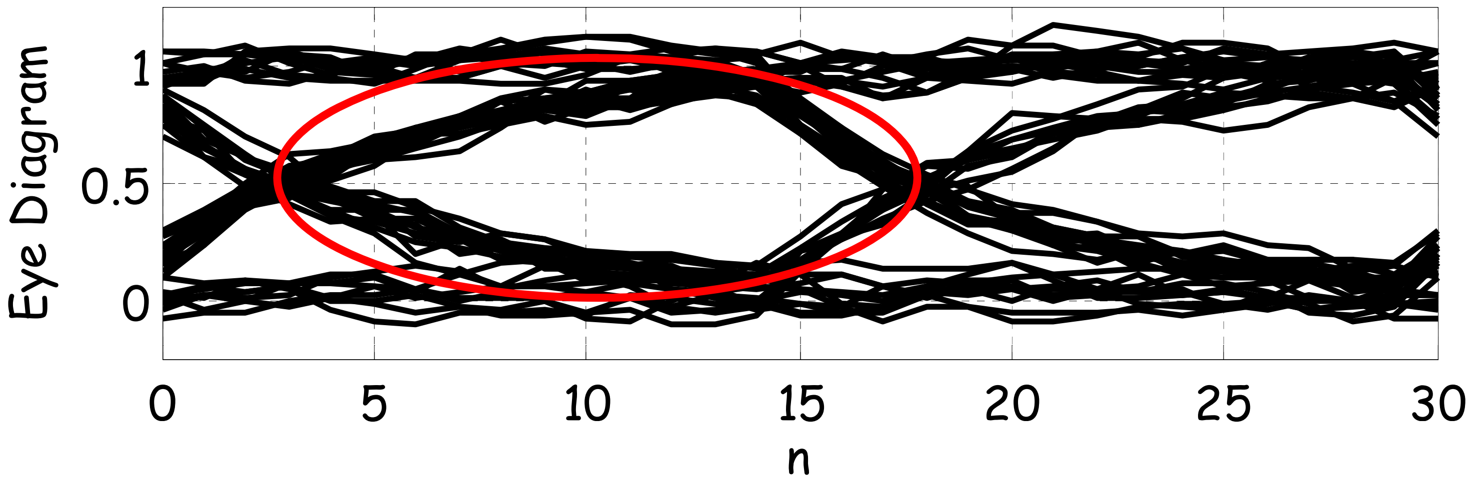




Channel: Exponential Step, a = 0.85, noise added

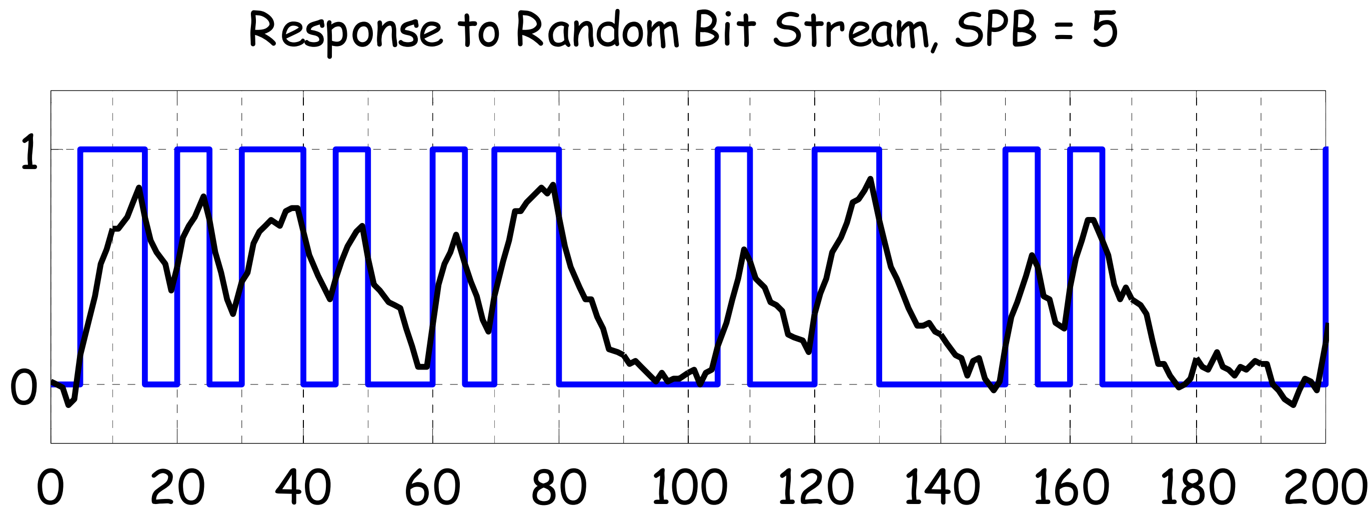
Eye Diagram (SPB = 15)





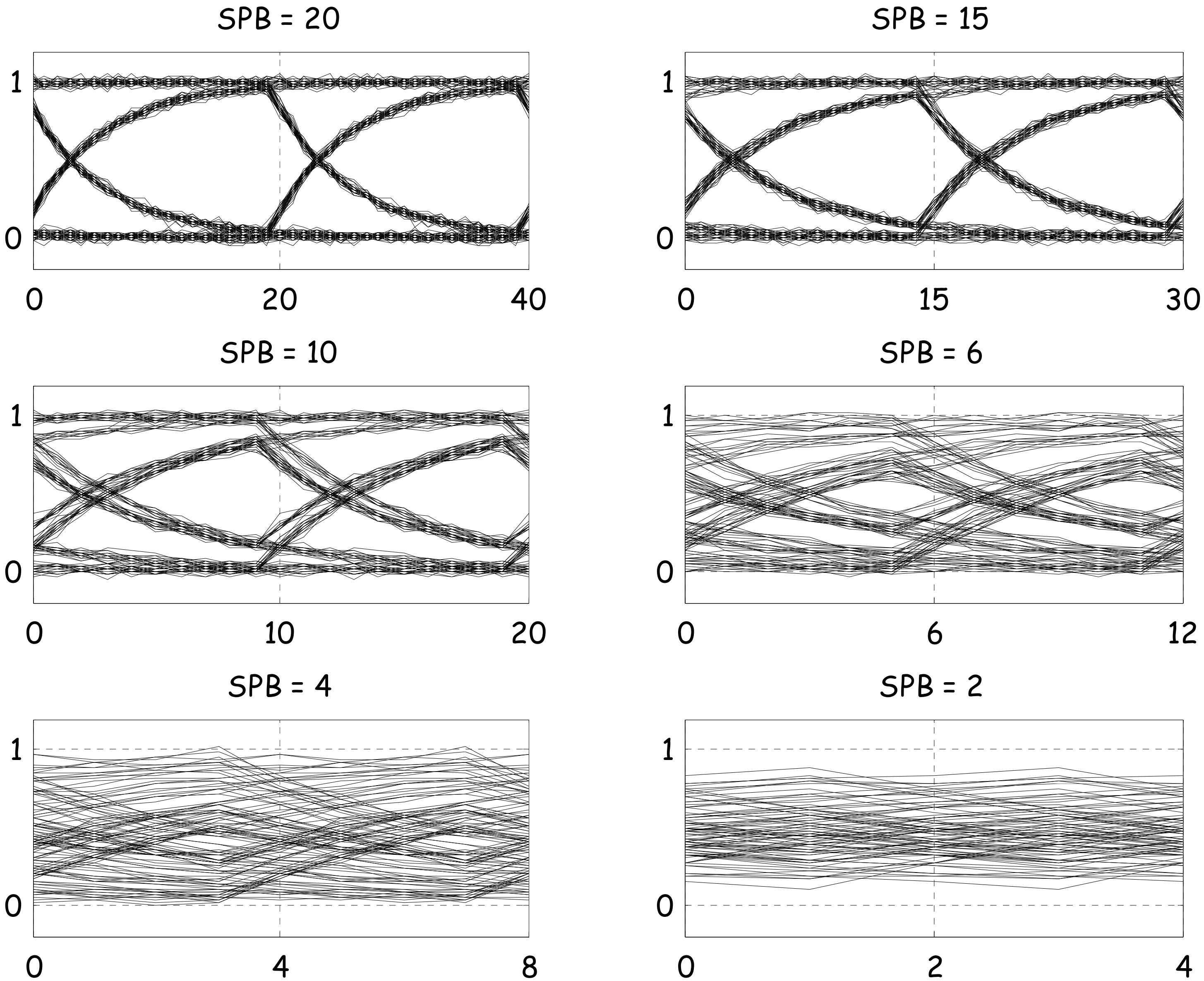
Channel: Exponential Step, a = 0.85, noise added

Eye Diagram (SPB = 5)



Channel: Exponential Step, a = 0.85, noise added

Eye Diagrams (Varying SPB)

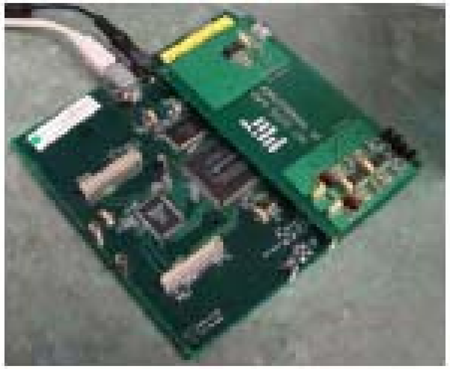


Channel: Exponential, a = 0.85, noise added

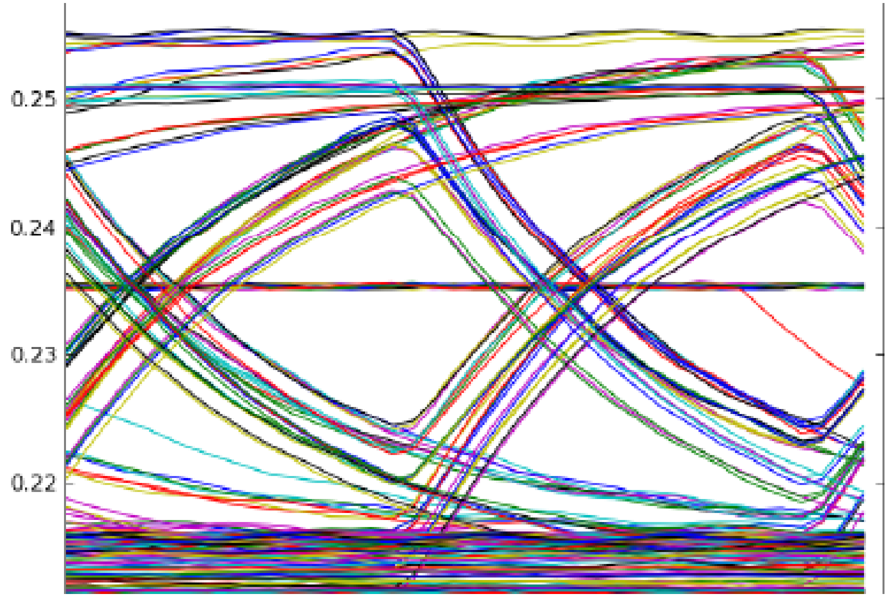
Observations

- As the bit time becomes shorter
  - The “height” of the eye gets smaller
    - “The eye begins to close”
  - The “width” of the eye gets narrower
    - The exact point to sample becomes more important
- Eventually, the eye closes completely, and there is no point you can sample at to avoid bit errors

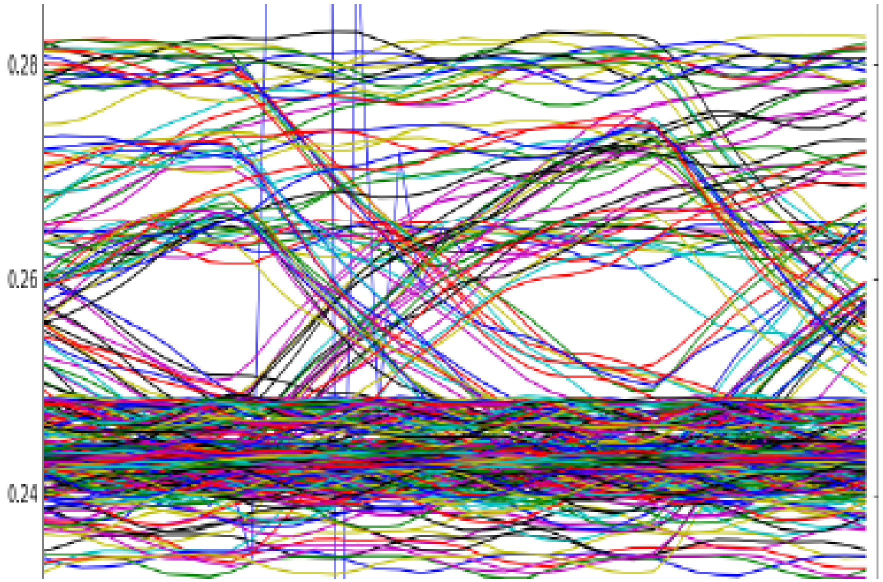
Eye Diagram from IR Channel



Eye Diagram 20 Samples per bit, dark room



Eye Diagram 20 Samples per bit, lights on!

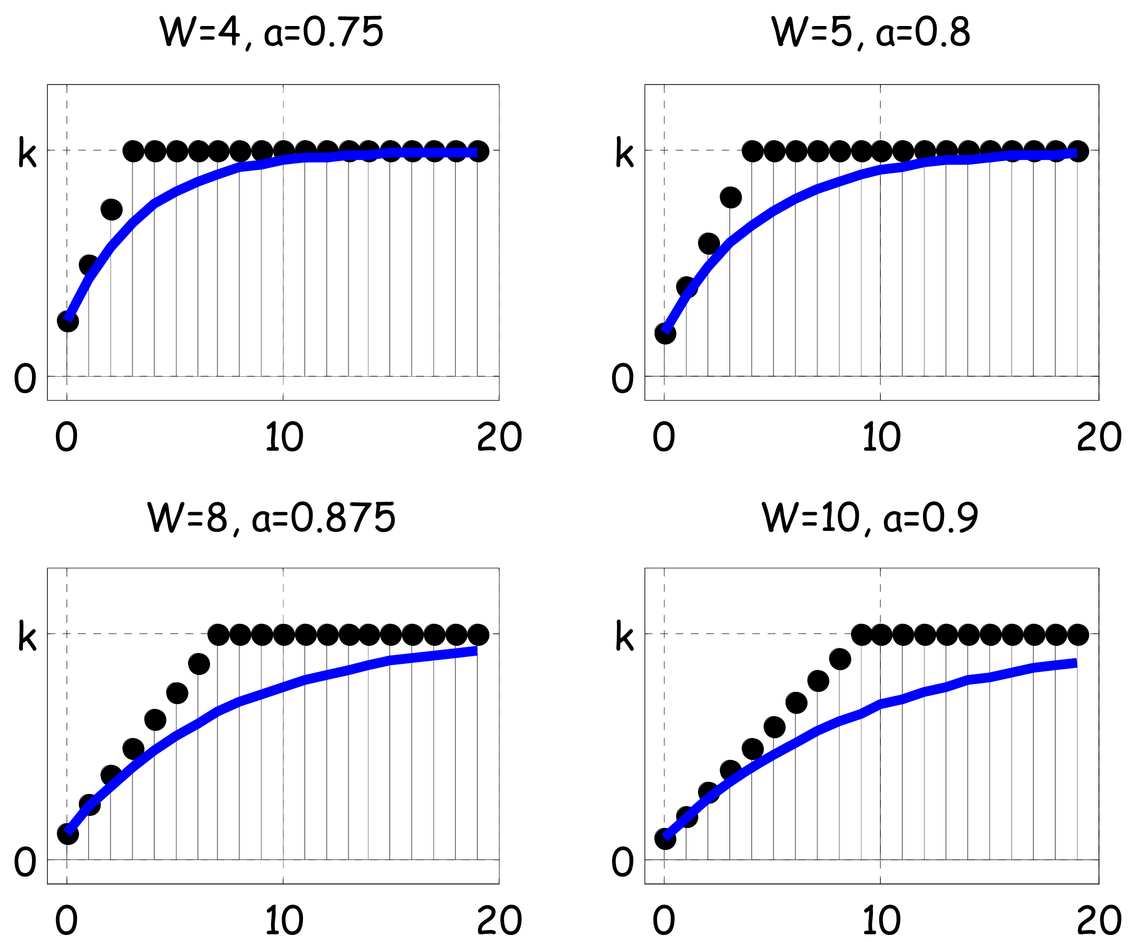


Next Time

- We will see how we can develop a different model of the channel.
- This model is equivalent to the step response, in the sense that it enables us to predict the output for any input.
- However, it will be expressed in a different way, using feedback.
- Later, we will see how this model enables us to "undo" the effect of the channel and "open" the eye

Saturating Ramp Step Response

- The saturating ramp is a simple approximation to the exponential response we studied earlier



Exponential:
$s(n) = k(1-a^{n+1})u(n)$

Saturating Ramp:

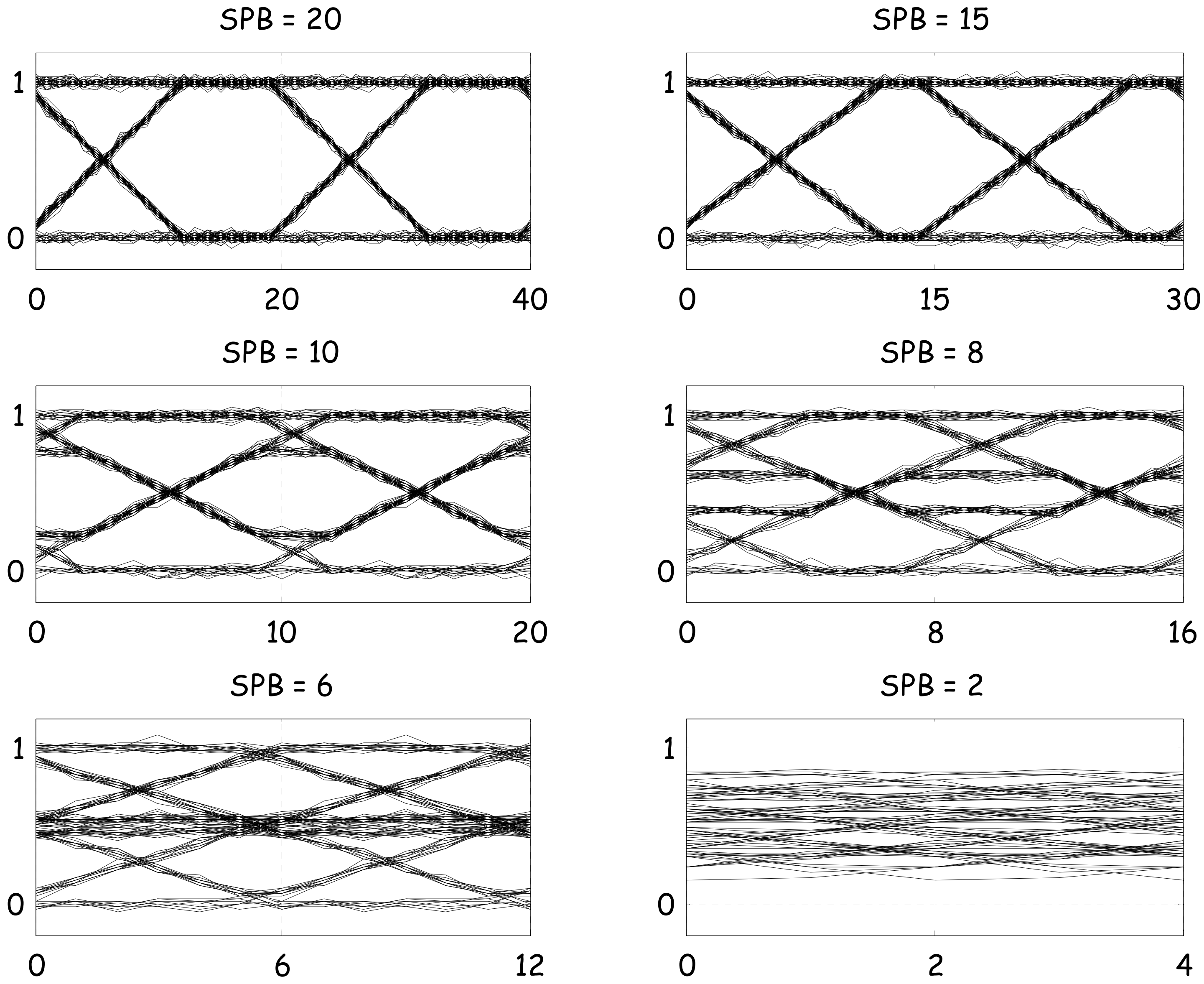
$$s(n) = \begin{cases} 0 & n < 0 \\ \dfrac{k(n+1)}{W} & 0 \le n < W \\ k & W \le n \end{cases}$$

For a "good" approximation
$a \approx 1 - \frac{1}{W}$

Properties of Saturating Ramp

- The value increases linearly to the final value.
- The transition takes W frames (from sample 0 to sample W-1).
- The transition is initially the same as the exponential step response, but the rate of increase of the exponential step response decreases over time, while that of the saturating ramp remains constant.
- Saturate = to reach a level beyond which you cannot further increase

Eye Diagrams (Varying SPB)



Channel: Saturated Ramp, W = 13, noise added