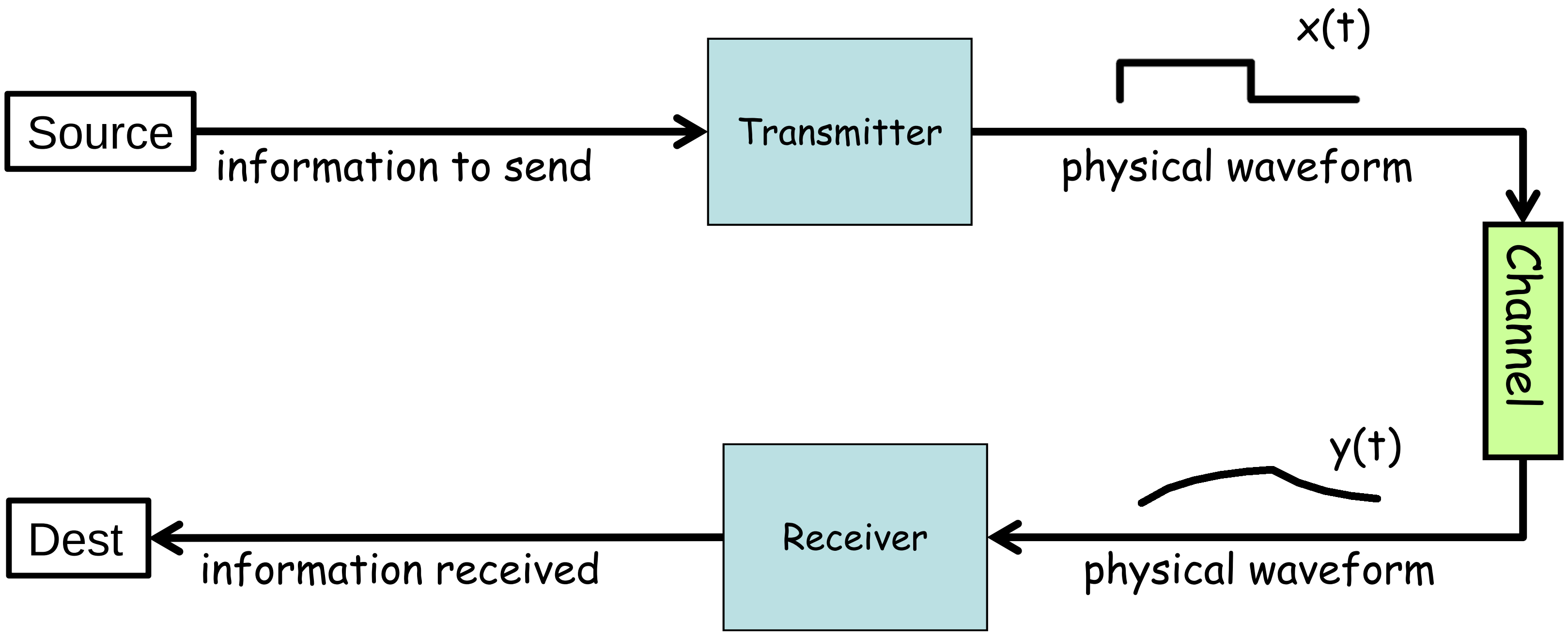
ELEC1200: A System View of Communications: from Signals to Packets Lecture 3

- Communication channels
  - Discrete time Channel
- Modeling the channel
  - Linear Time Invariant Systems
  - Step Response
  - Response to single bit
  - Response to general input

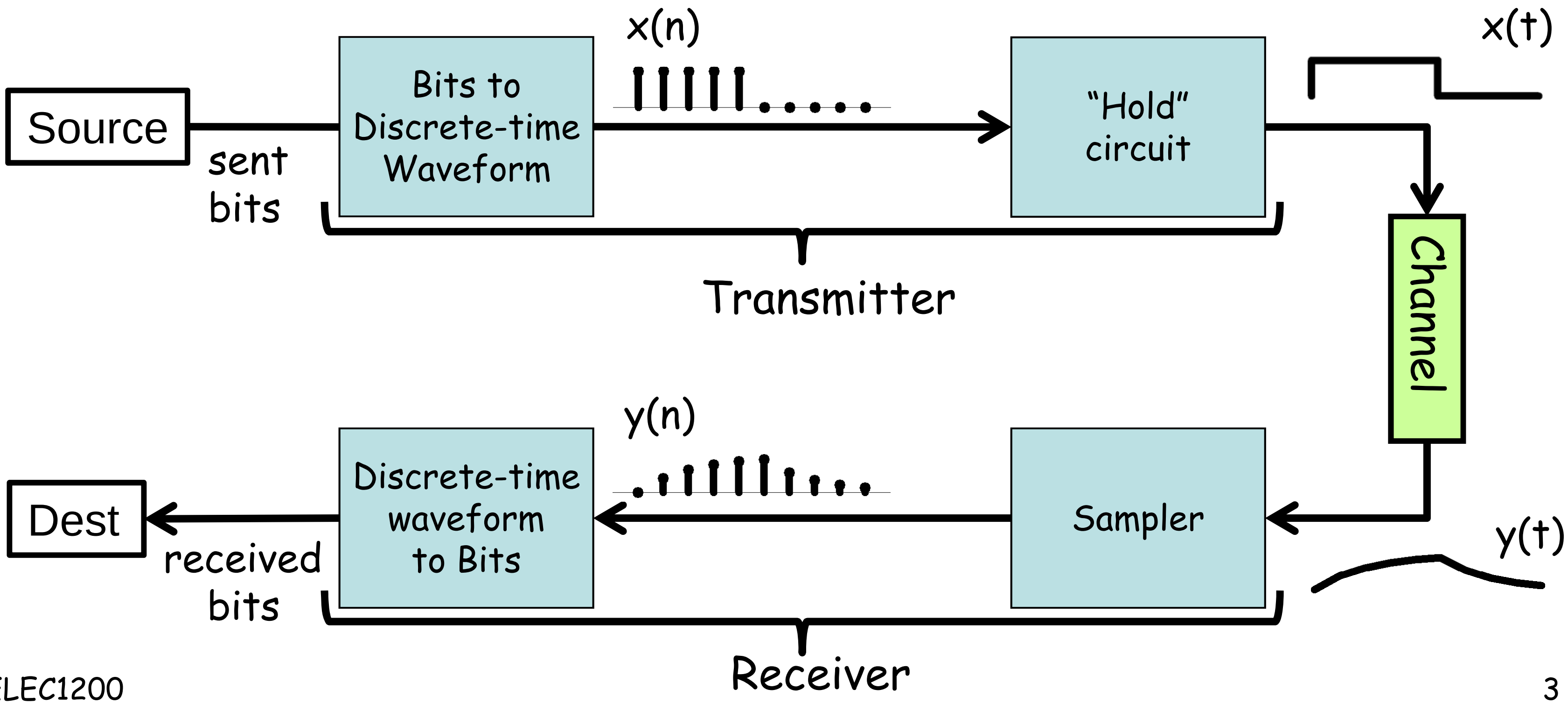
Communication System

- Transmitter encodes information as a physical waveform.
- This physical waveform travels over a medium (e.g. air), which modifies the waveform as it passes. This medium is called the channel.
- The receiver must take this modified waveform and try to figure out the original information



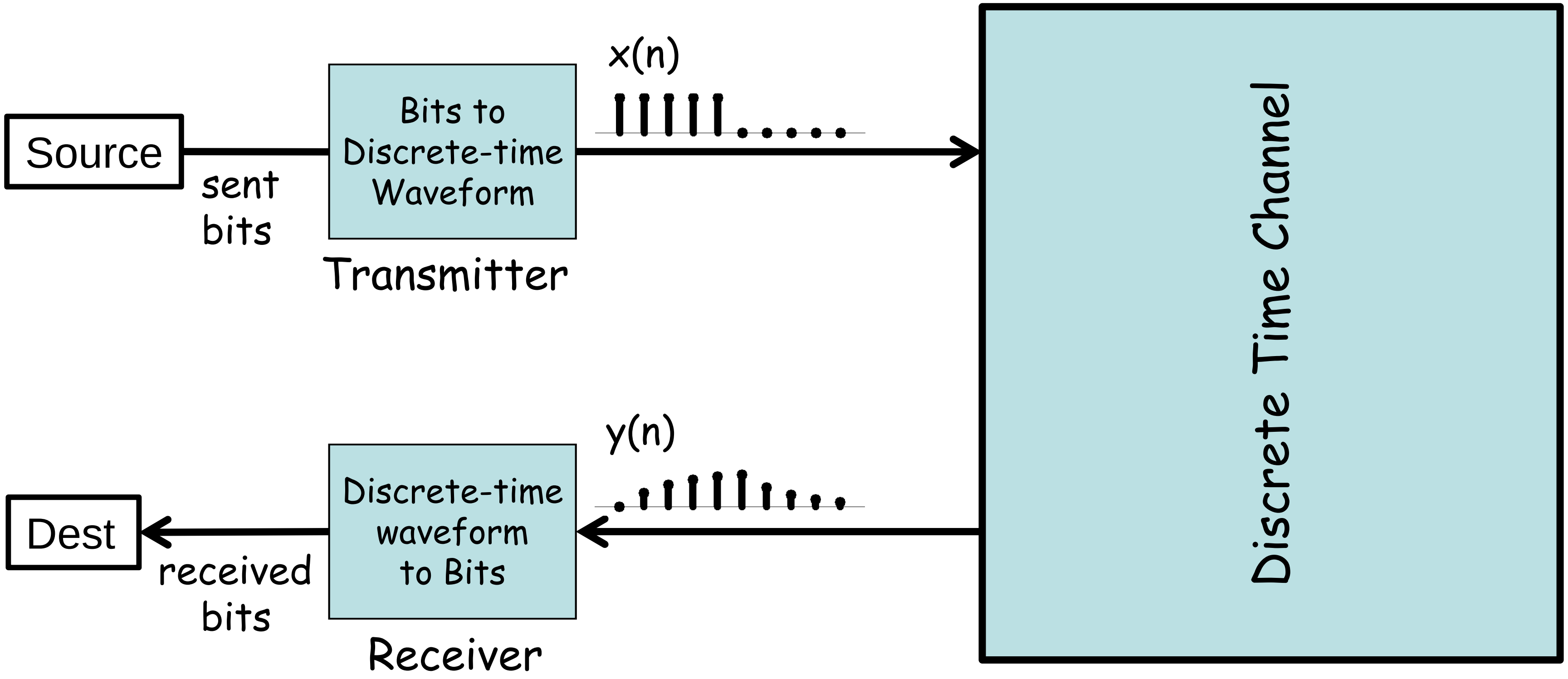
Discrete Time Channel

- Modern communication systems are built from computers that work in discrete time, but most channels are continuous time.
- Thus, modern transmitters create discrete time signals, and then generate continuous time signals to send.
- Modern receivers receive continuous time signals, which they sample to obtain discrete time signals.



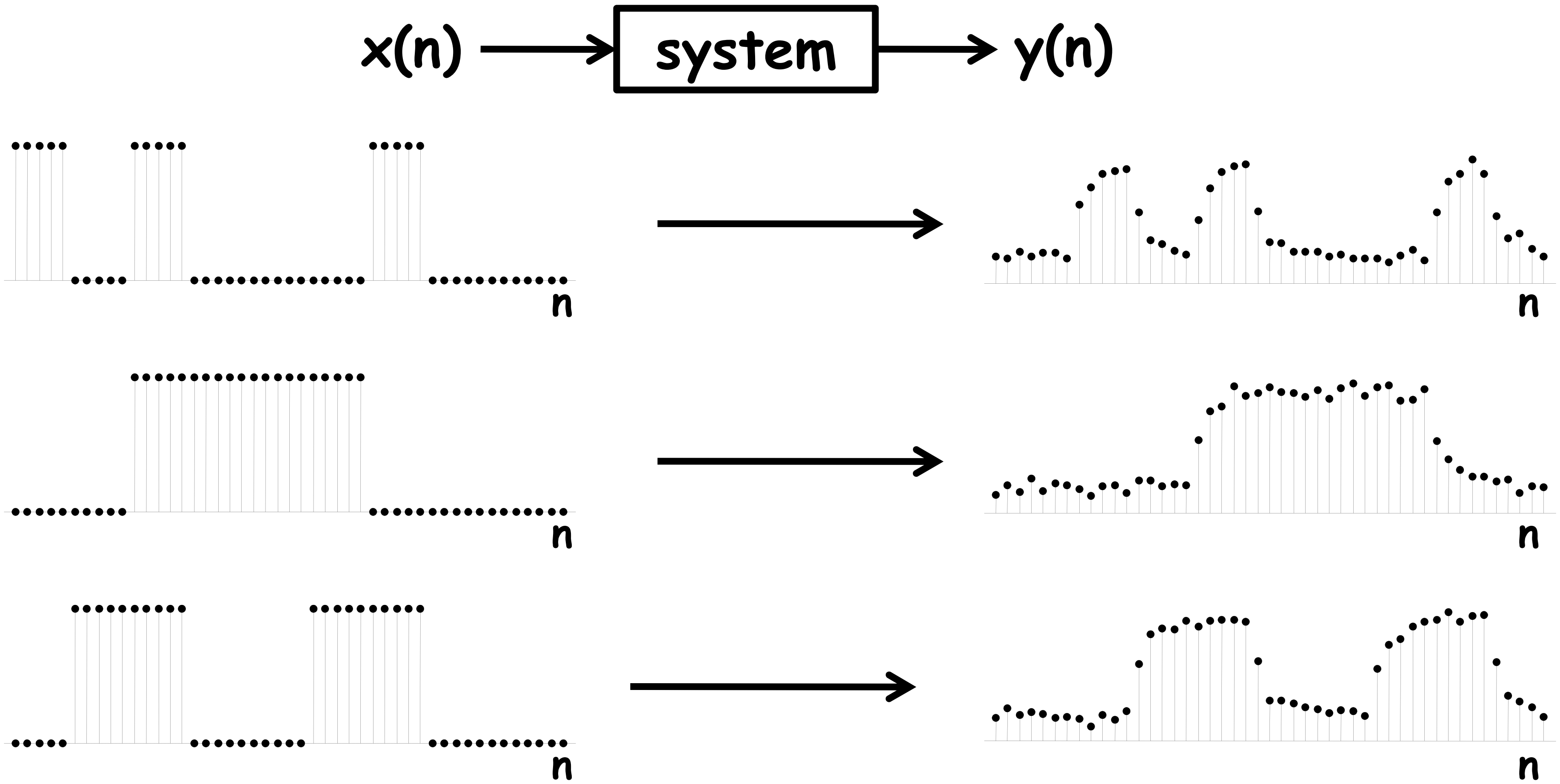
Discrete Time Channel

- For this course, we will not worry about the details of the discrete/continuous conversion. We treat the mapping from the transmitted samples x(n) to the received samples y(n) as a "black box", called a discrete time channel.
- Definition: A "black box" is something we cannot see the insides of, just what comes in and out, like a magician's hat.



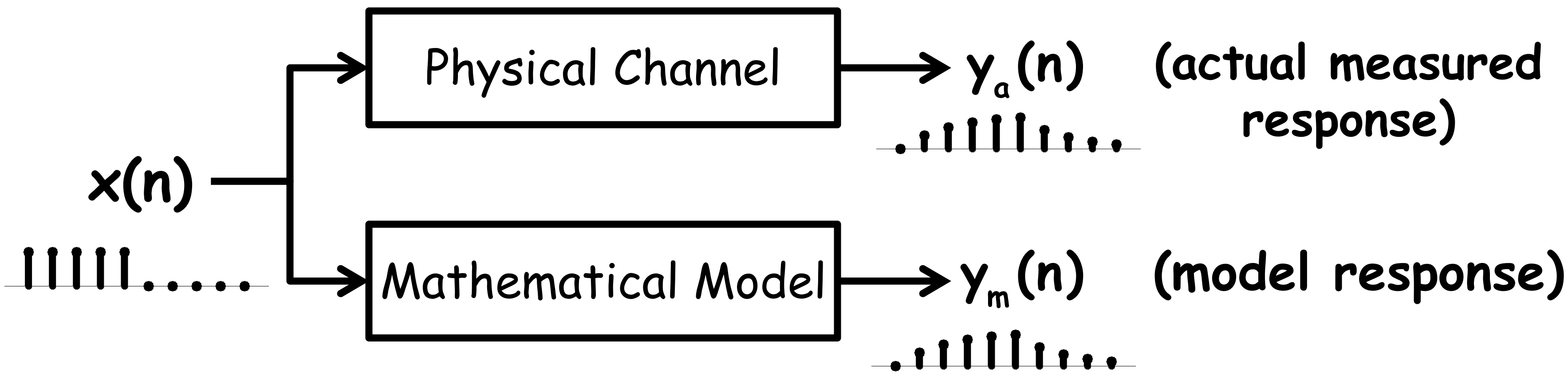
Systems

- For this class, a <u>system</u> is something that takes a waveform x(n) and produces an output waveform y(n) (e.g. a channel)



Mathematical Models

- A mathematical model describes the operation of a system using mathematical formulas.

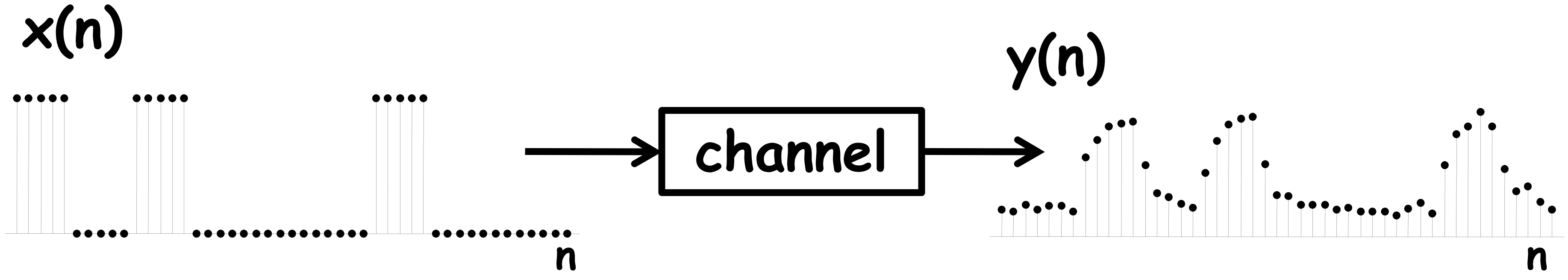


- We have a good model if
  - The model and actual responses are similar, $y_m(n) \approx y_a(n)$
  - The relationship between $y_m(n)$ and $x(n)$ is simple.
- Engineers use mathematical models to
  - Understand the operation of the system
  - Predict the performance of a system in different situations
  - Develop modifications to the system that improve performance.
  - Design new systems for new applications.

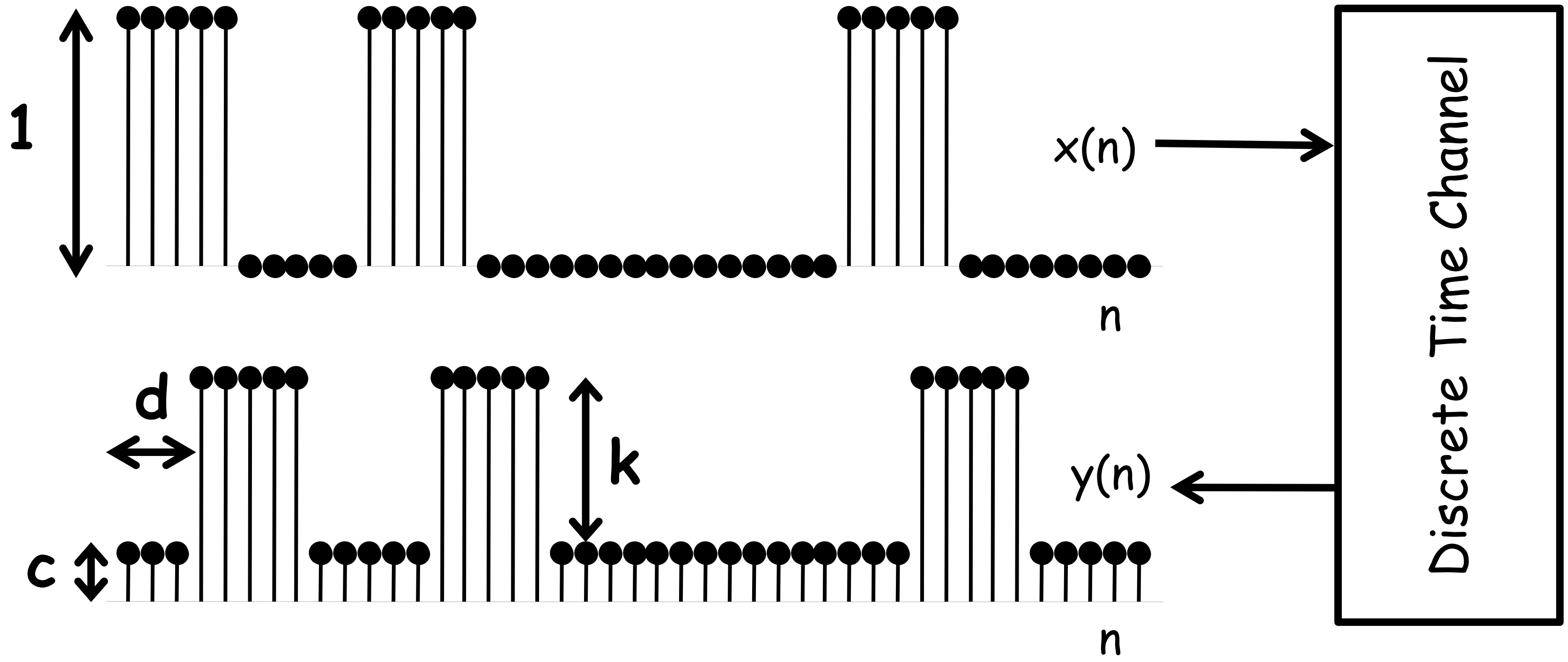
Possible effects of the channel

- The channel may cause the received signal y(n) to differ from the transmitted signal x(n) in several ways, including
  - Attenuation (decrease in amplitude)
  - Delay
  - Offset
  - Blurring of transitions
  - Noise

we study only these four now



Modelling attenuation, delay, and offset



- k = attenuation (k < 1)
- d = delay
- c = offset

What are possible physical causes for these?

- Mathematical Model:

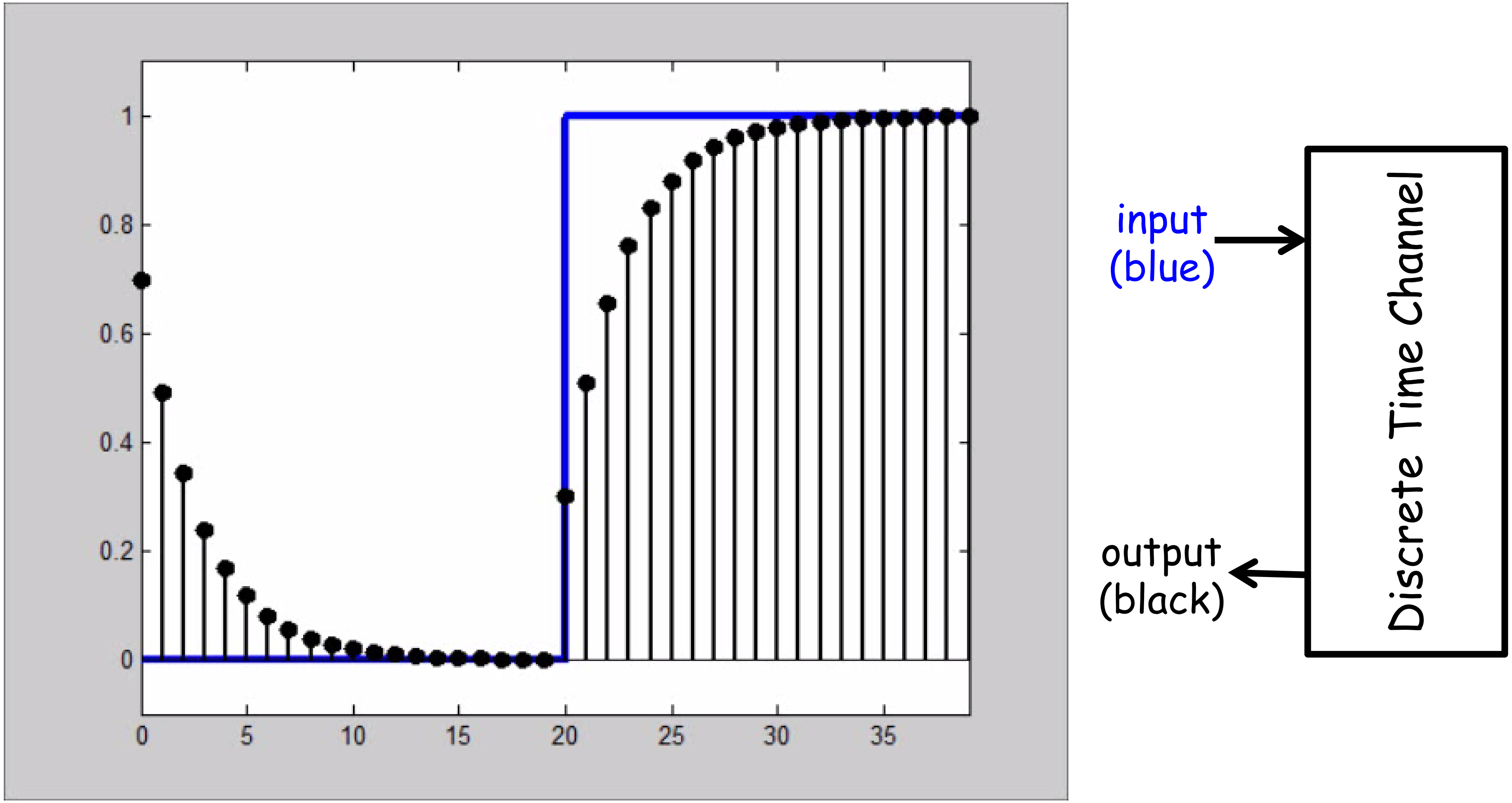
$$y(n) = kx(n - d) + c$$

Blurring of transitions

- The bit sequence is encoded by a waveform that makes instantaneous changes from one value (1) to another (0).
- Due to physical limitations in
  - The transducer that creates the physical waveform
  - The electronics that drive the transducer
  - The physical medium that carries the waveform
  - The sensor that senses the physical waveform
  - The electronics that process the sensor signal

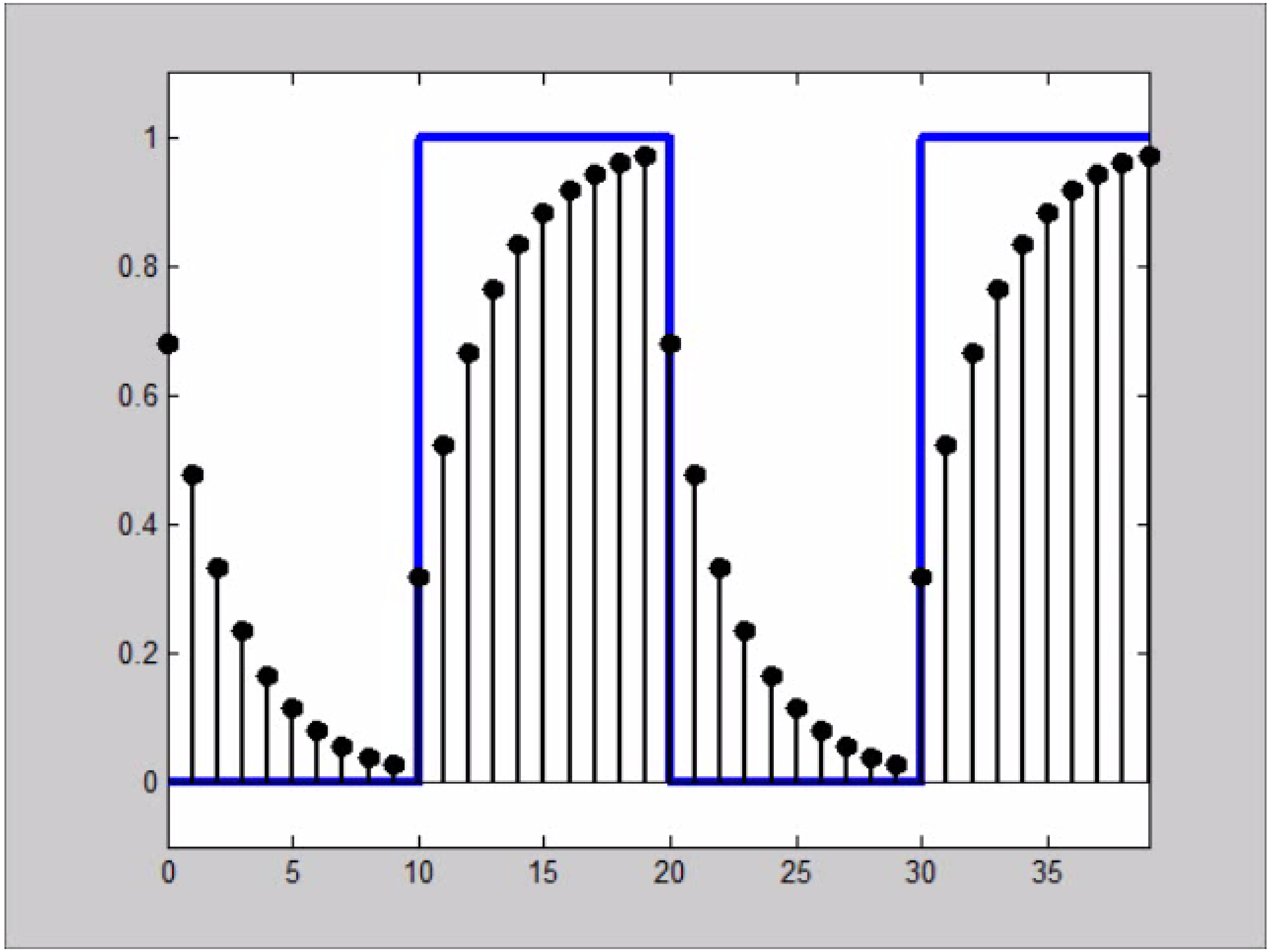
  the actual received waveform cannot make instantaneous changes.
- In engineering terms, we say that the channel is <u>bandlimited</u>.

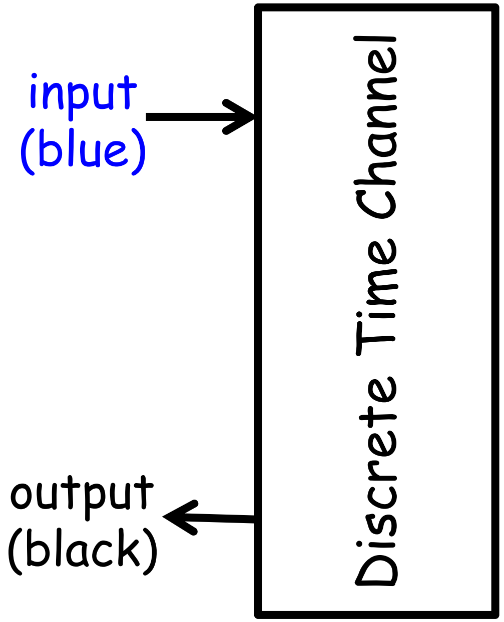
Effect of bandlimited channel (SPB=20)



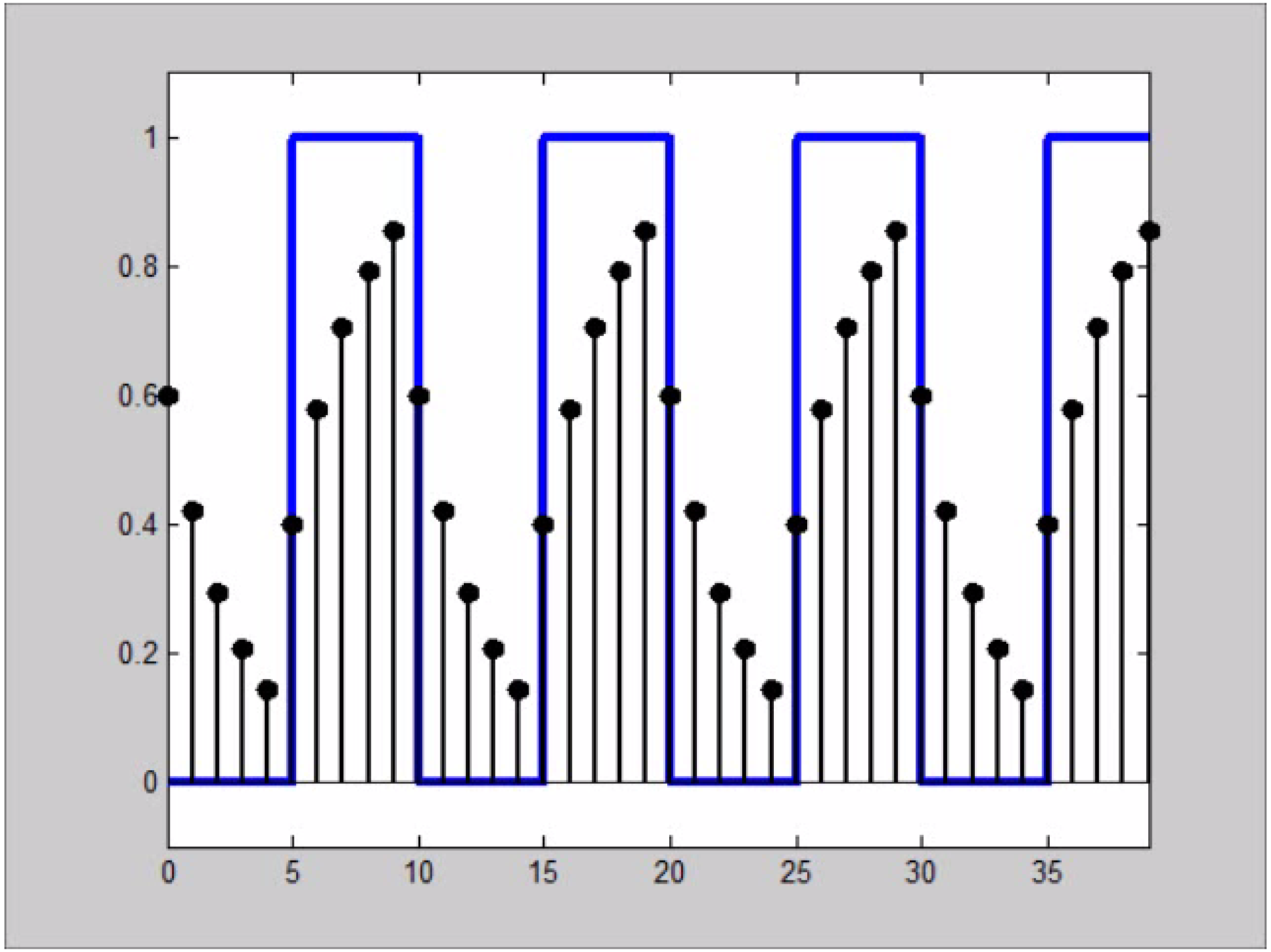
Assumes no attenuation, delay, offset or noise.

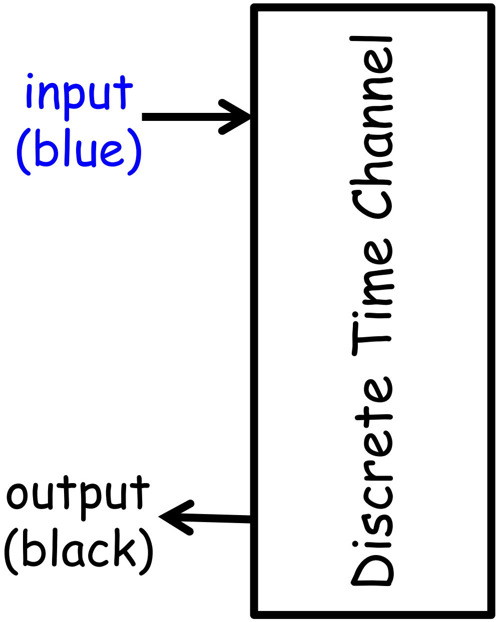
Effect of bandlimited channel (SPB=10)





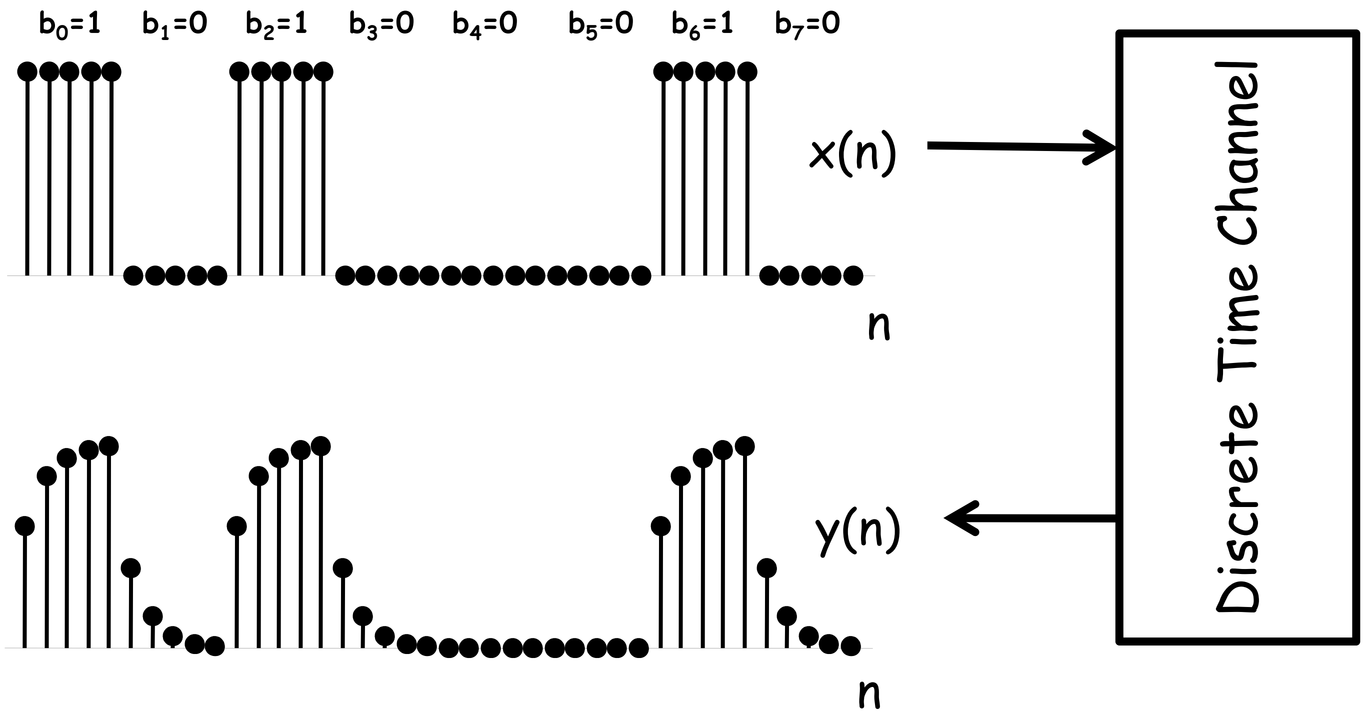
Effect of bandlimited channel (SPB=5)





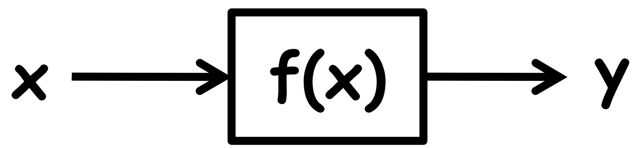
Developing a bandlimited channel model

- Can we develop a mathematical model that enables us to predict the output of a bandlimited channel to any input?
- Yes! If we assume that the channel is <u>linear</u> and <u>time invariant</u>.
- We use the fact that any input can be expressed as the sum of unit step functions.

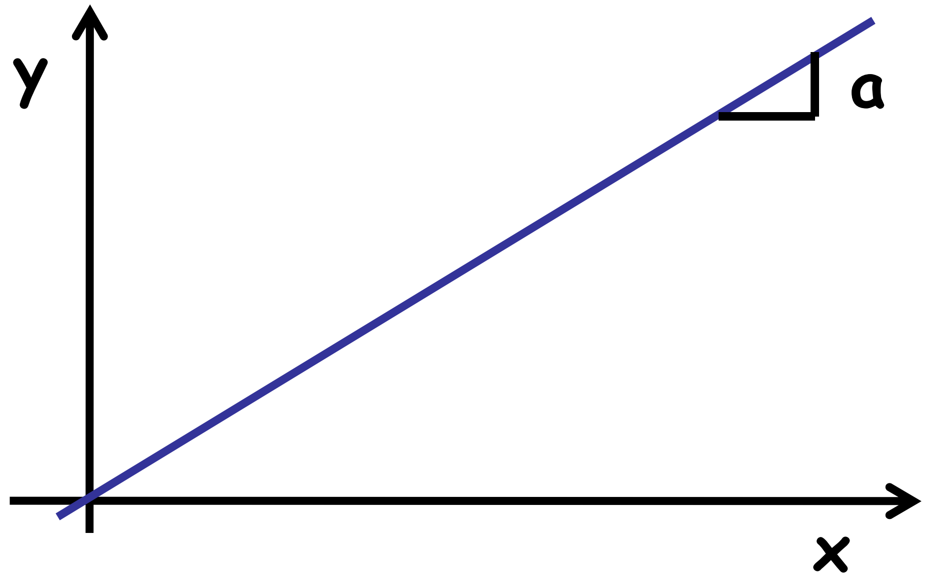


Linear Functions

- A <u>function</u> y = f(x) can be viewed as something that takes a numerical input (x) and produces a numerical output (y)



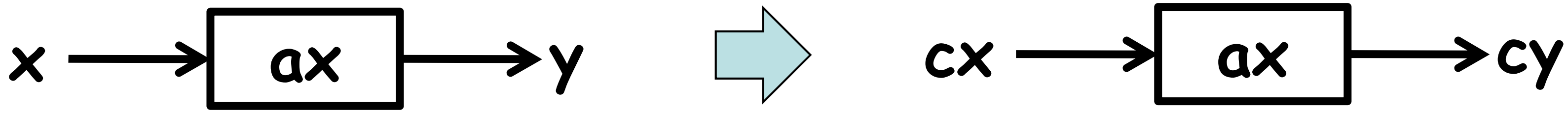
- A <u>linear function</u> is a function of the form y = ax where a is a constant.



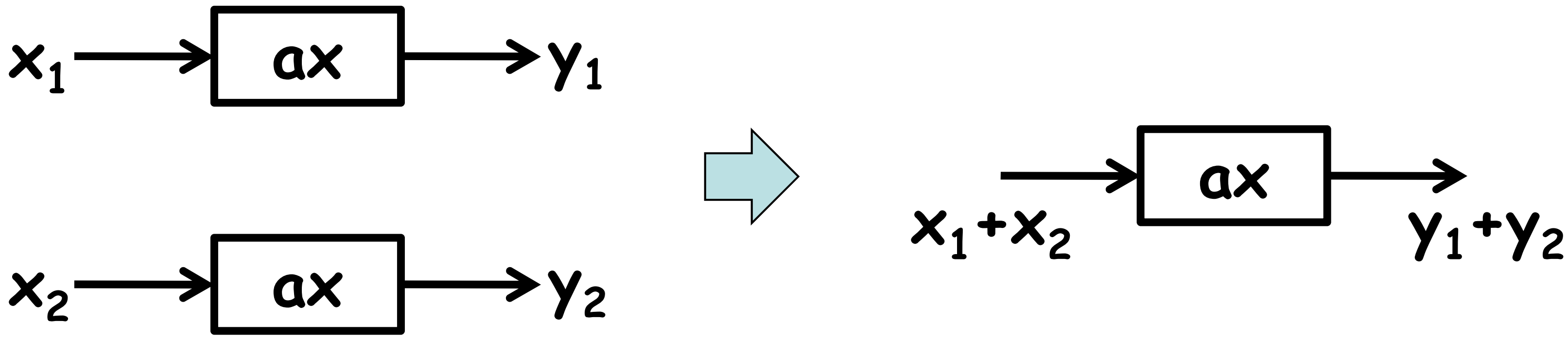
- Note: y = ax+b is <u>not</u> linear (unless b=0).

Properties of Linear Functions

- Homogeneity: If you multiply the input by a constant, c, you get the output multiplied by c.



- Additivity: If you put the sum of two numbers into the function, the output is the sum of the outputs to each number applied (put in) to the function individually.



Nice thing about linear functions

- If you know
  - a function is linear, and
  - the output for <u>any</u> nonzero input

  then, you can compute the output for any other input.
- Example:

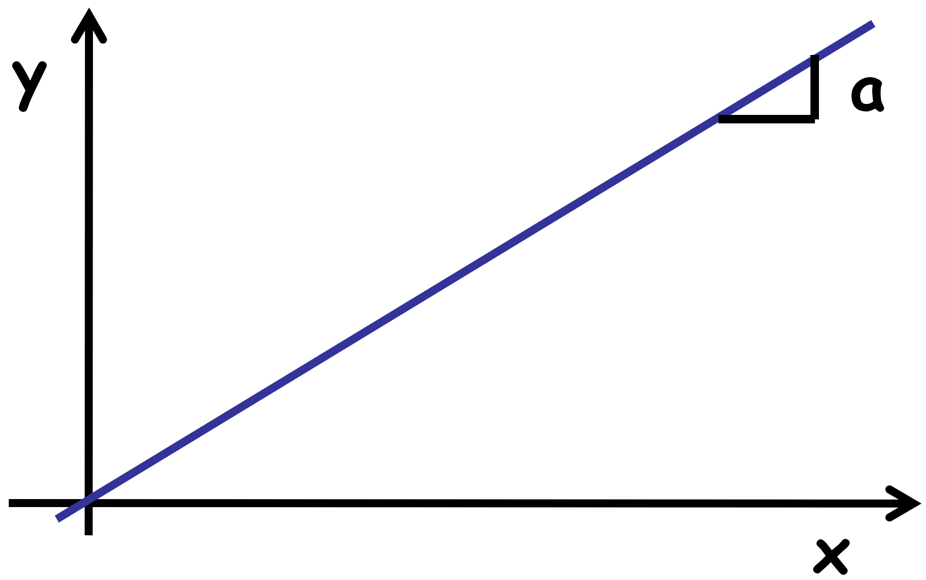
  The output of a linear function for $x = 2$ is $y = 4$.

  What is the output if

  $x = 4$?

  $x = 6$?

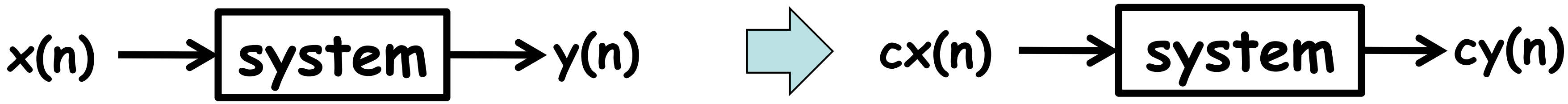
  $x = 8.5$?



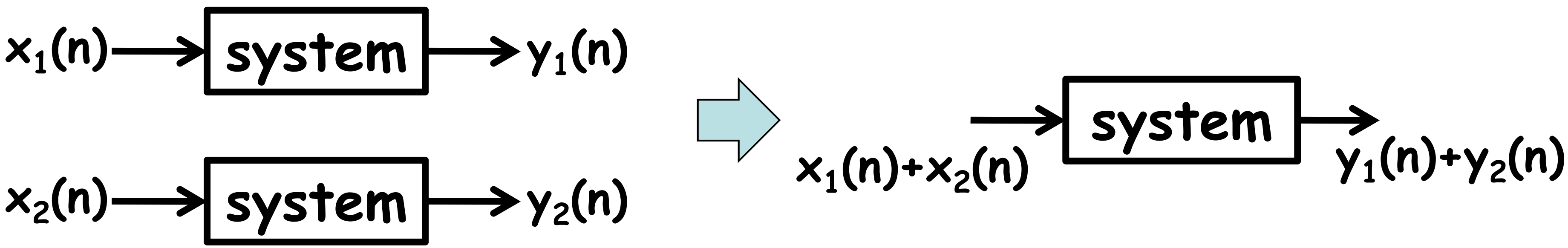
Linear Systems

- A <u>linear system</u> is a system that satisfies the same two properties as a linear function.

- <u>Homogeneity</u>:

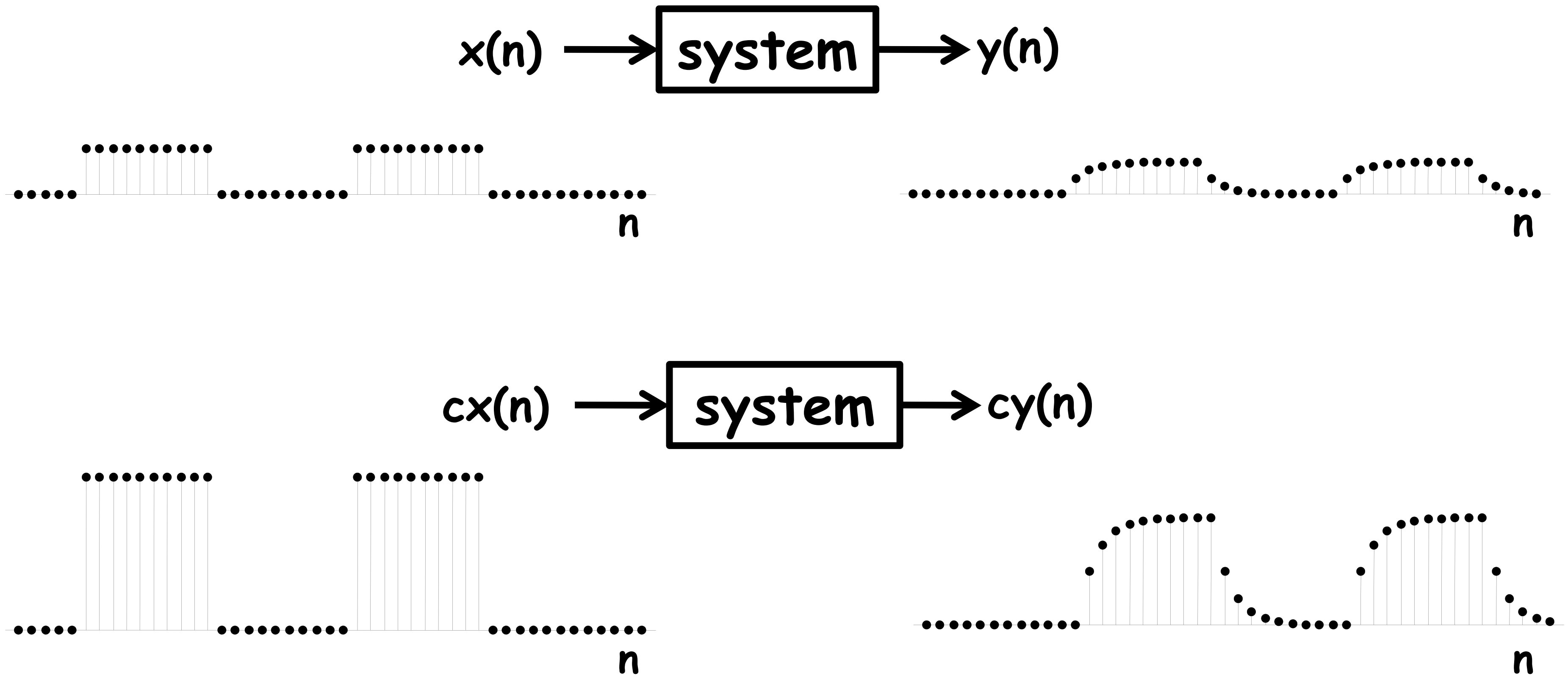


- <u>Additivity</u>



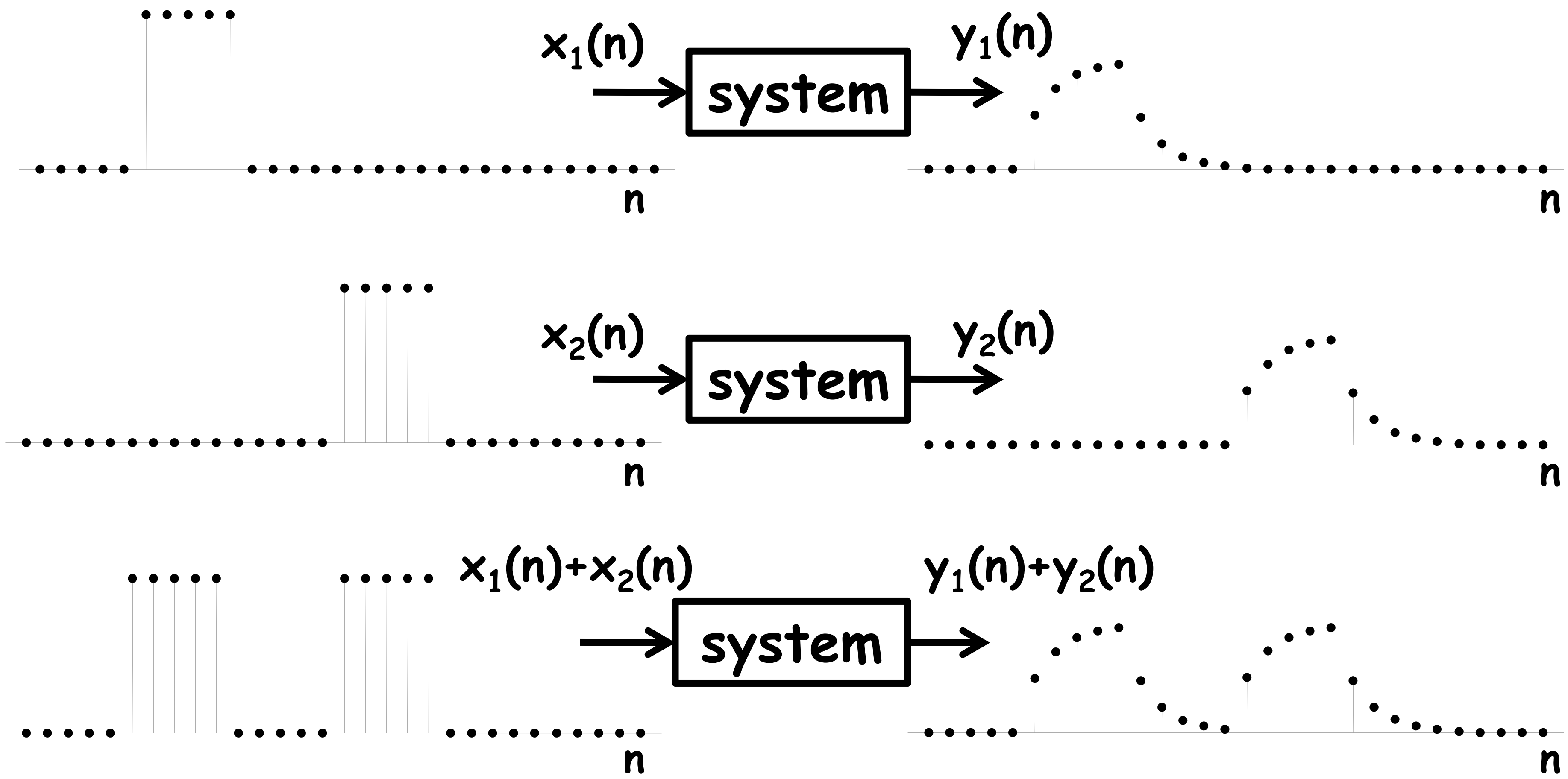
Homogeneity

- If you scale (multiply) the input by c times, the output is also scaled by c times.



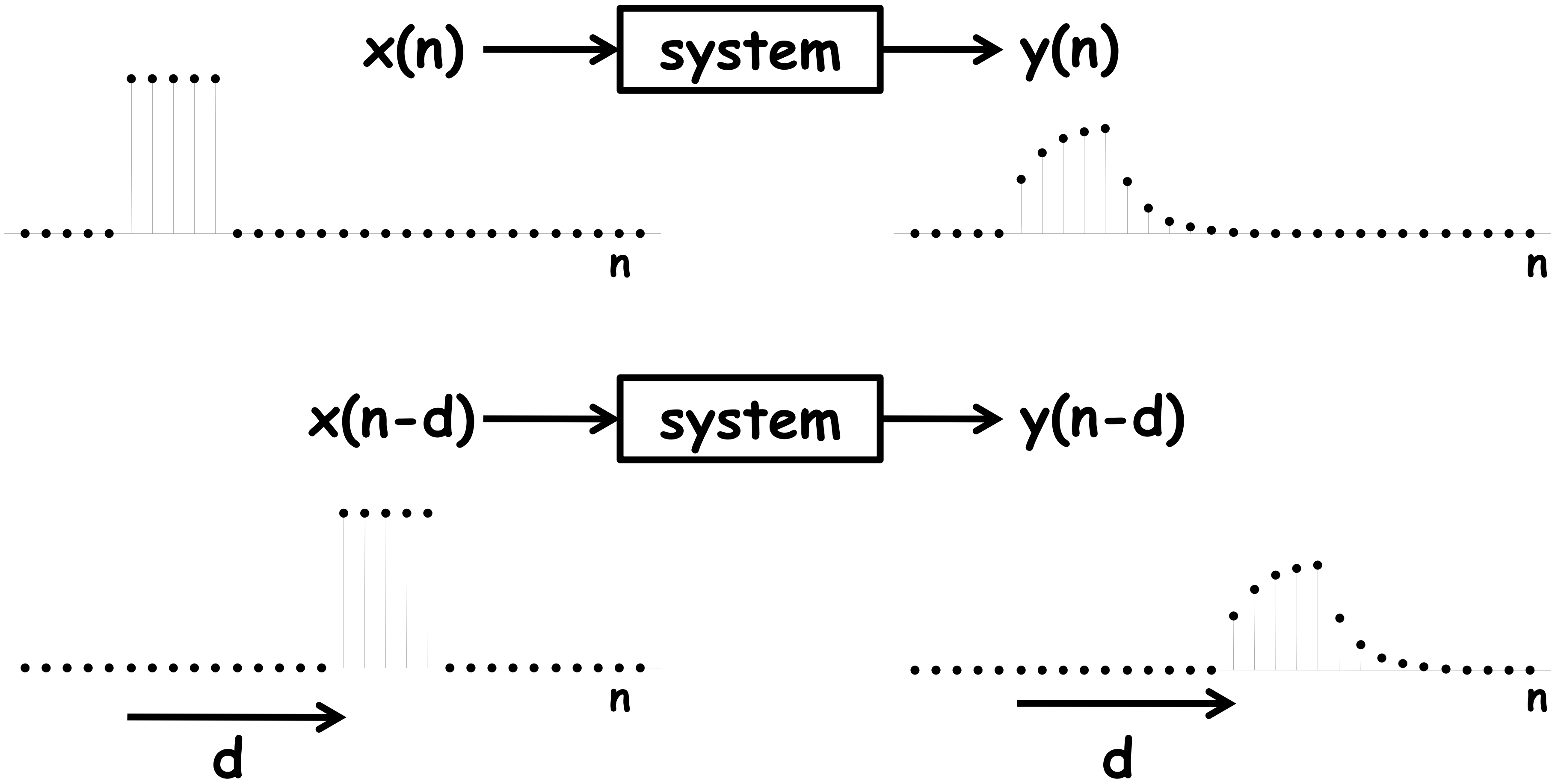
Additivity

- The output to the sum of two inputs is the sum of the outputs to each input considered individually.



Time Invariant Systems

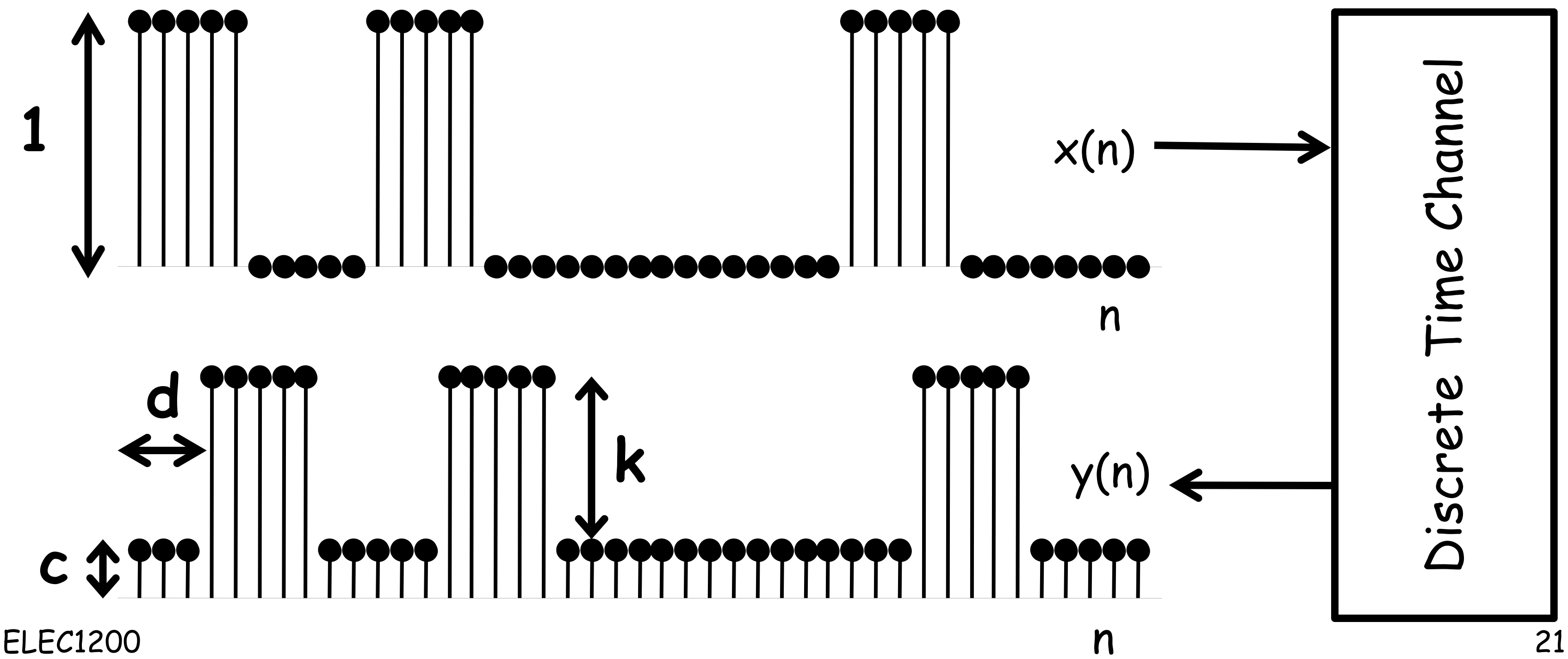
- A <u>time invariant system</u> is one where if you delay the input by d, you get the exact same output, just delayed by the same amount d.



Linear Time Invariant (LTI) Systems

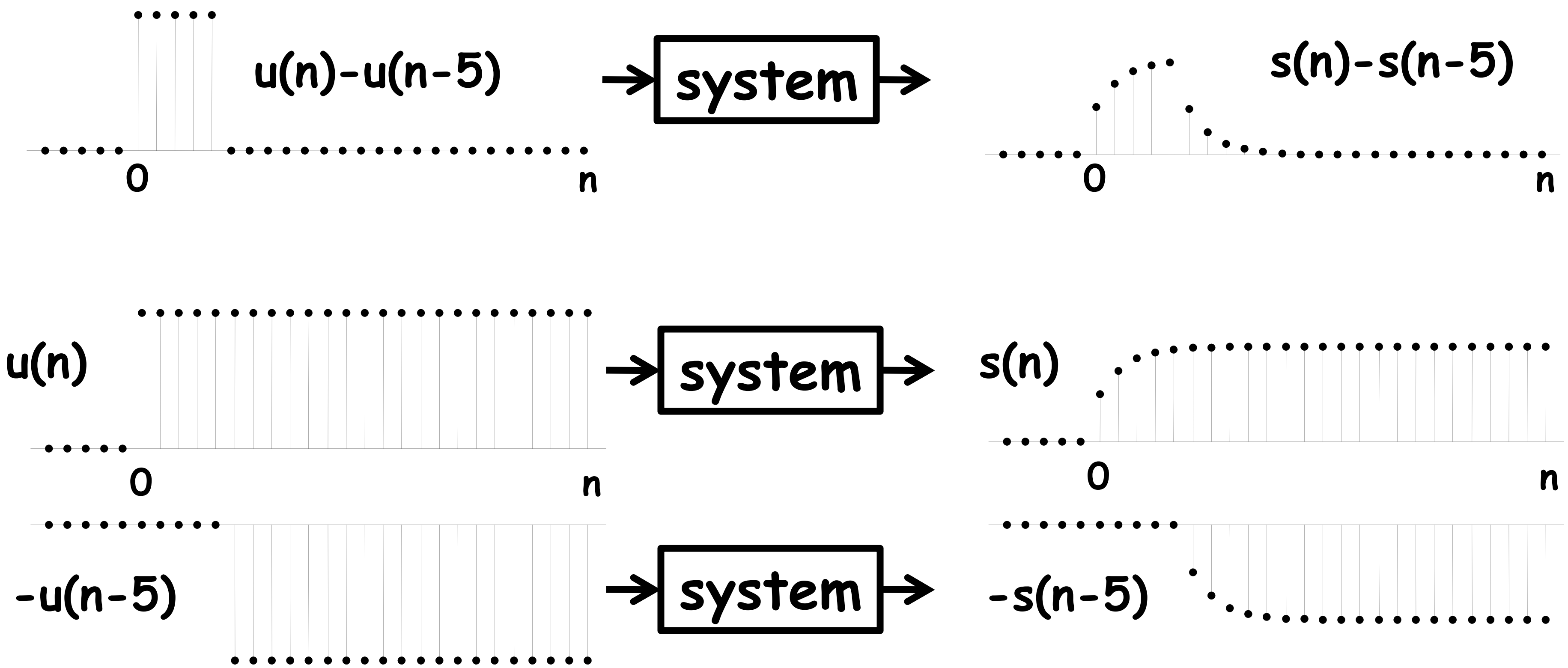
- A system that is both linear and time invariant is known as an LTI system
- Is the channel with attenuation, delay and offset an LTI system?

$$y(n) = kx(n-d) + c$$



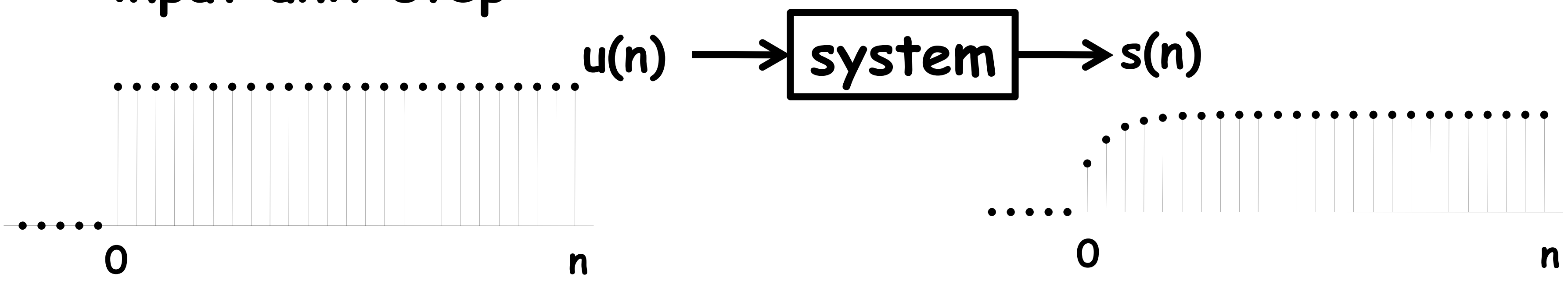
Output of LTI Systems

- If a system is LTI, then you can find the output just by knowing the output to a unit step function.
- How? Express the input as the sum/difference of unit step functions and use linearity and time invariance



Step Response s(n)

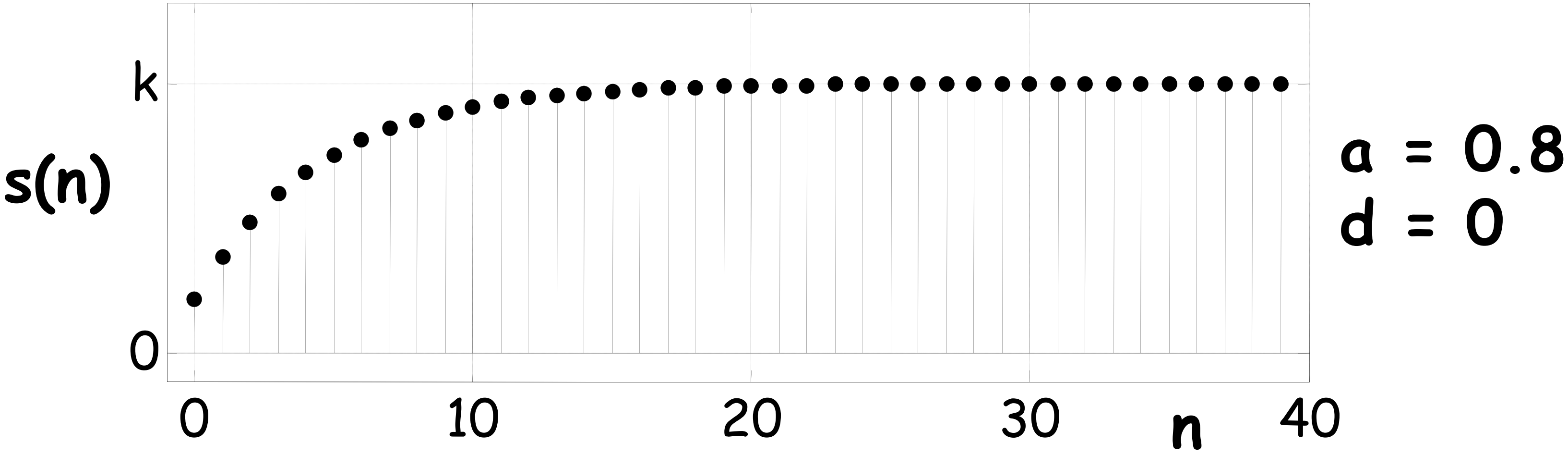
- <u>step response</u> = the output of the system to a input unit step

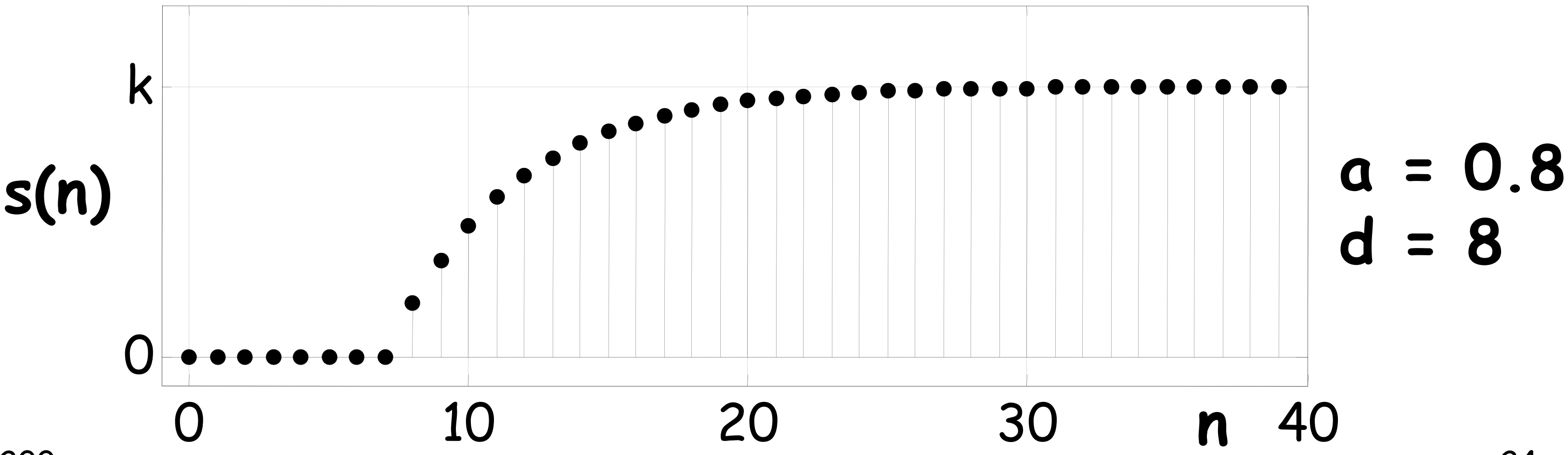


- The exponential step response (next page) is a good model for the step responses of the communication channel in our lab.
- It can model the following effects:
  - changes in amplitude (k)
  - propagation delay (d)
  - blurring of transitions (a)

Exponential Step Response

$$s(n) = k(1 - a^{n-d+1})u(n-d)$$





Exponential Step Response

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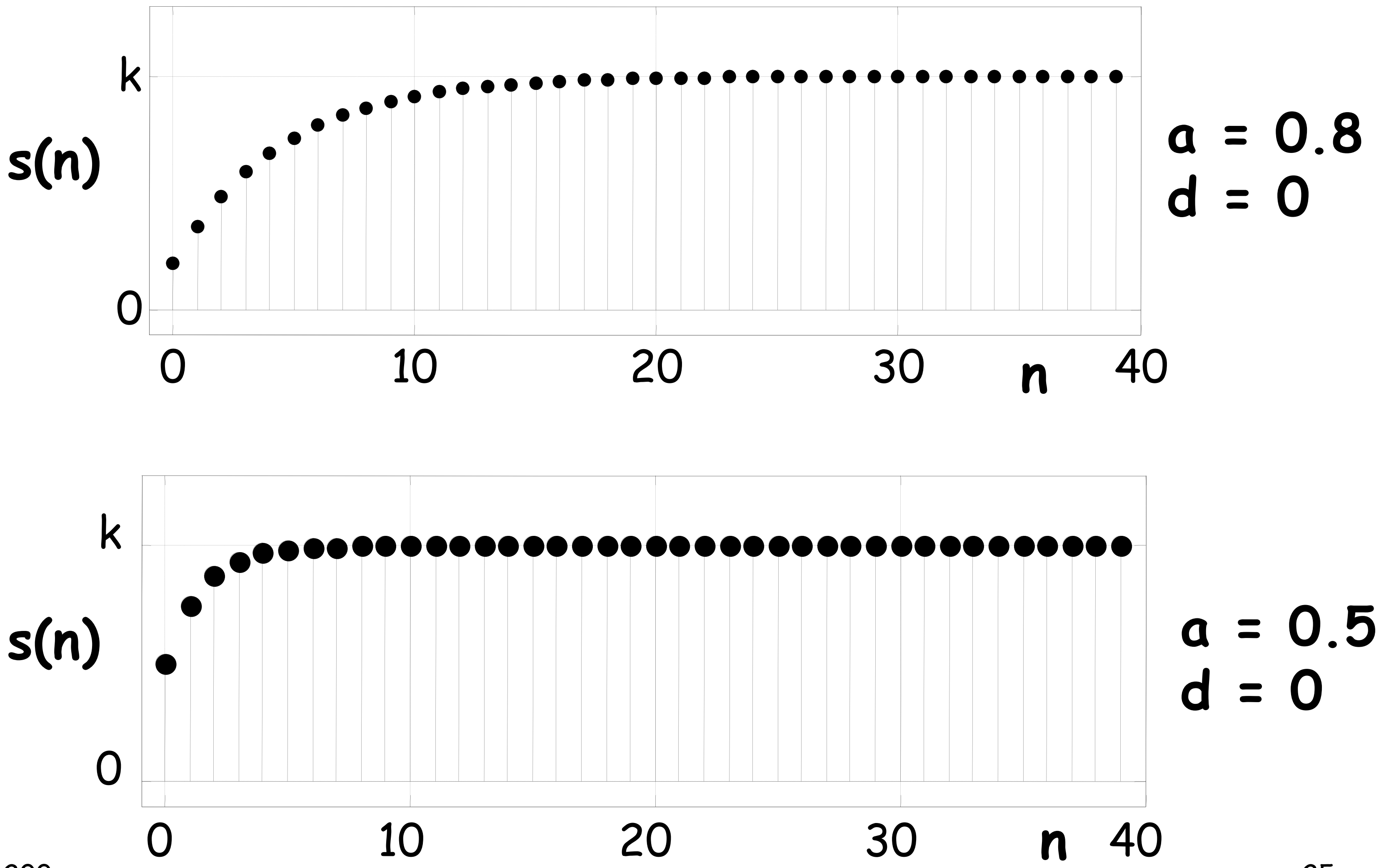
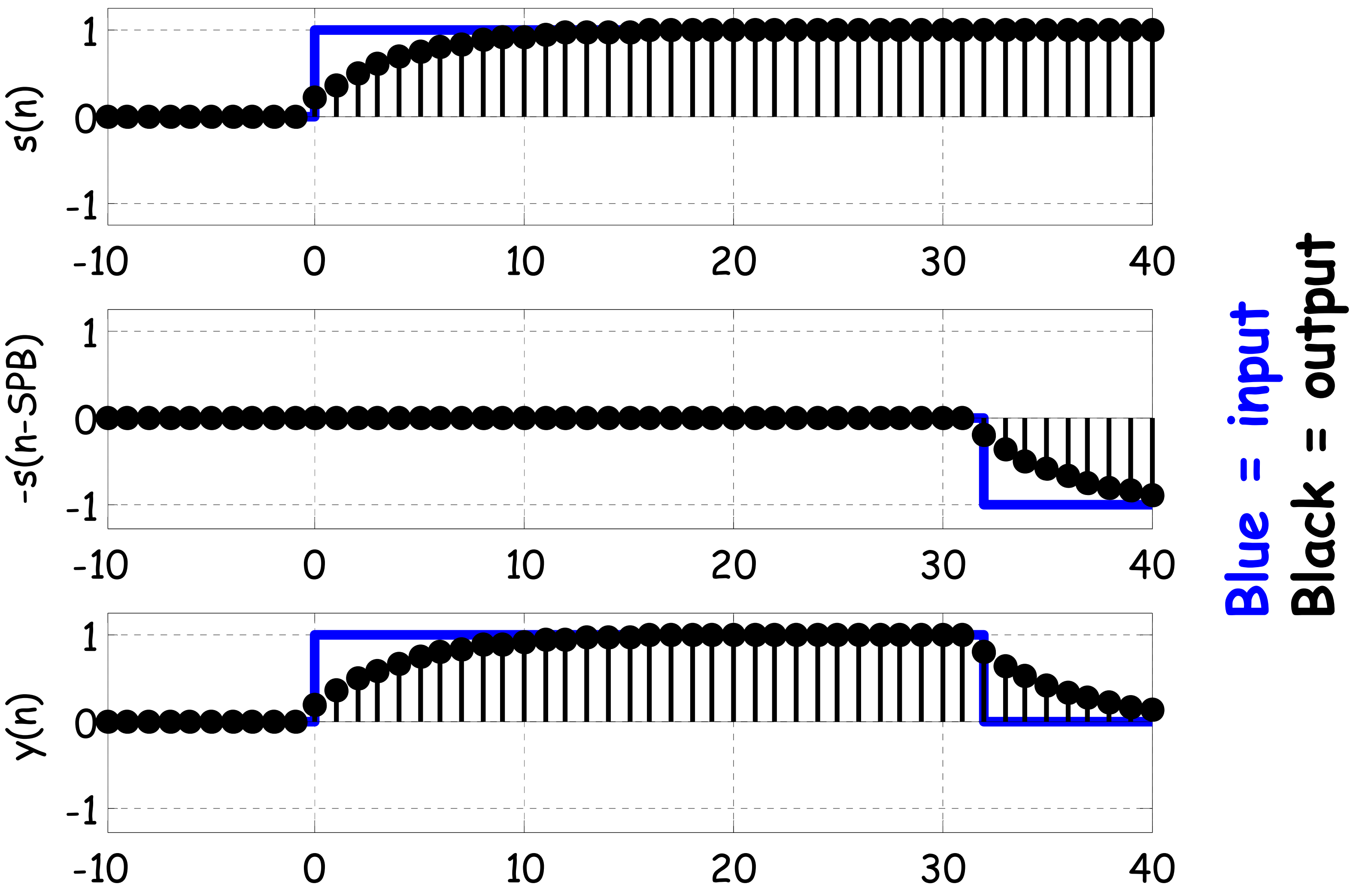
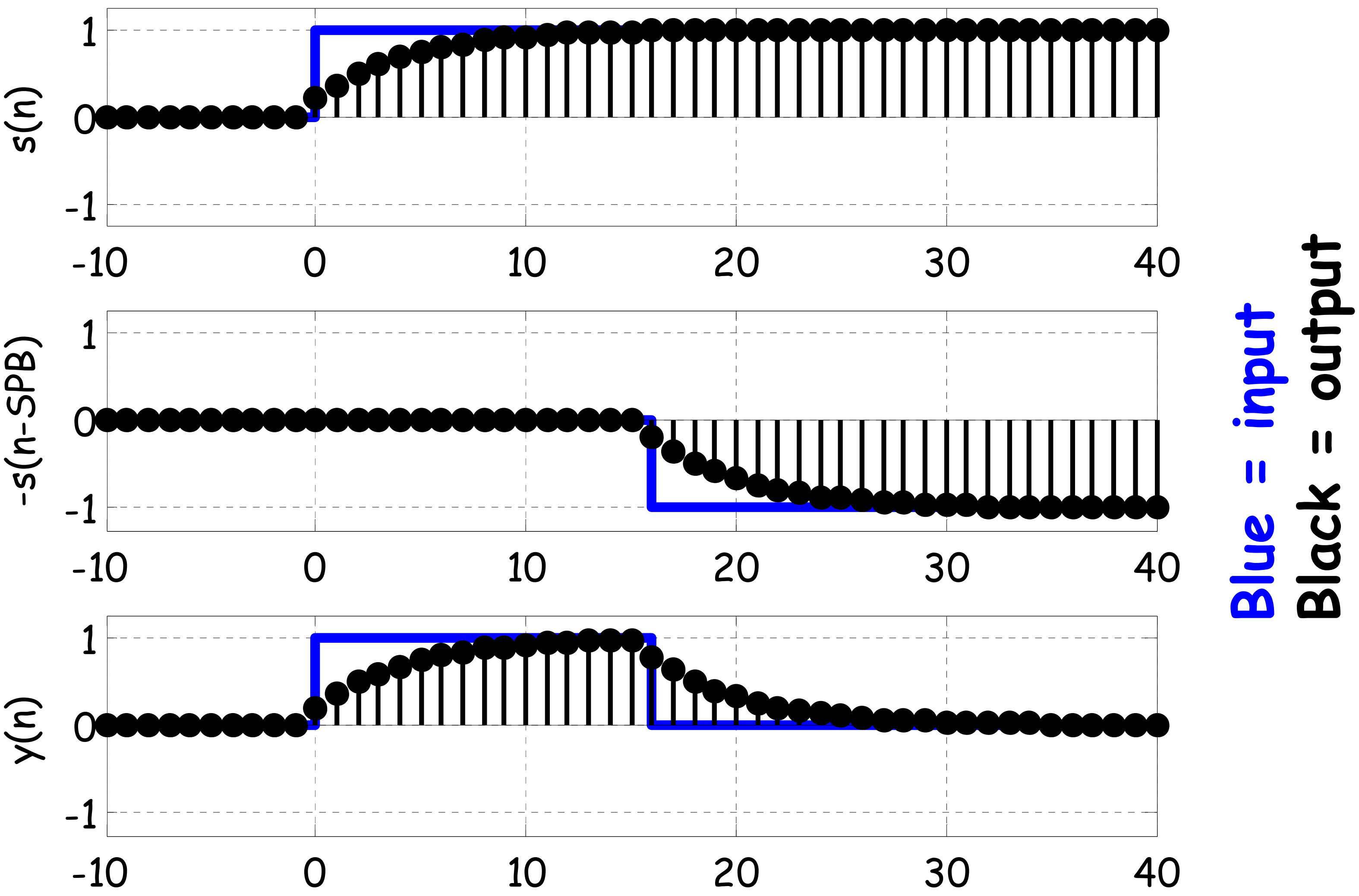


Response to single bit (SPB = 32)



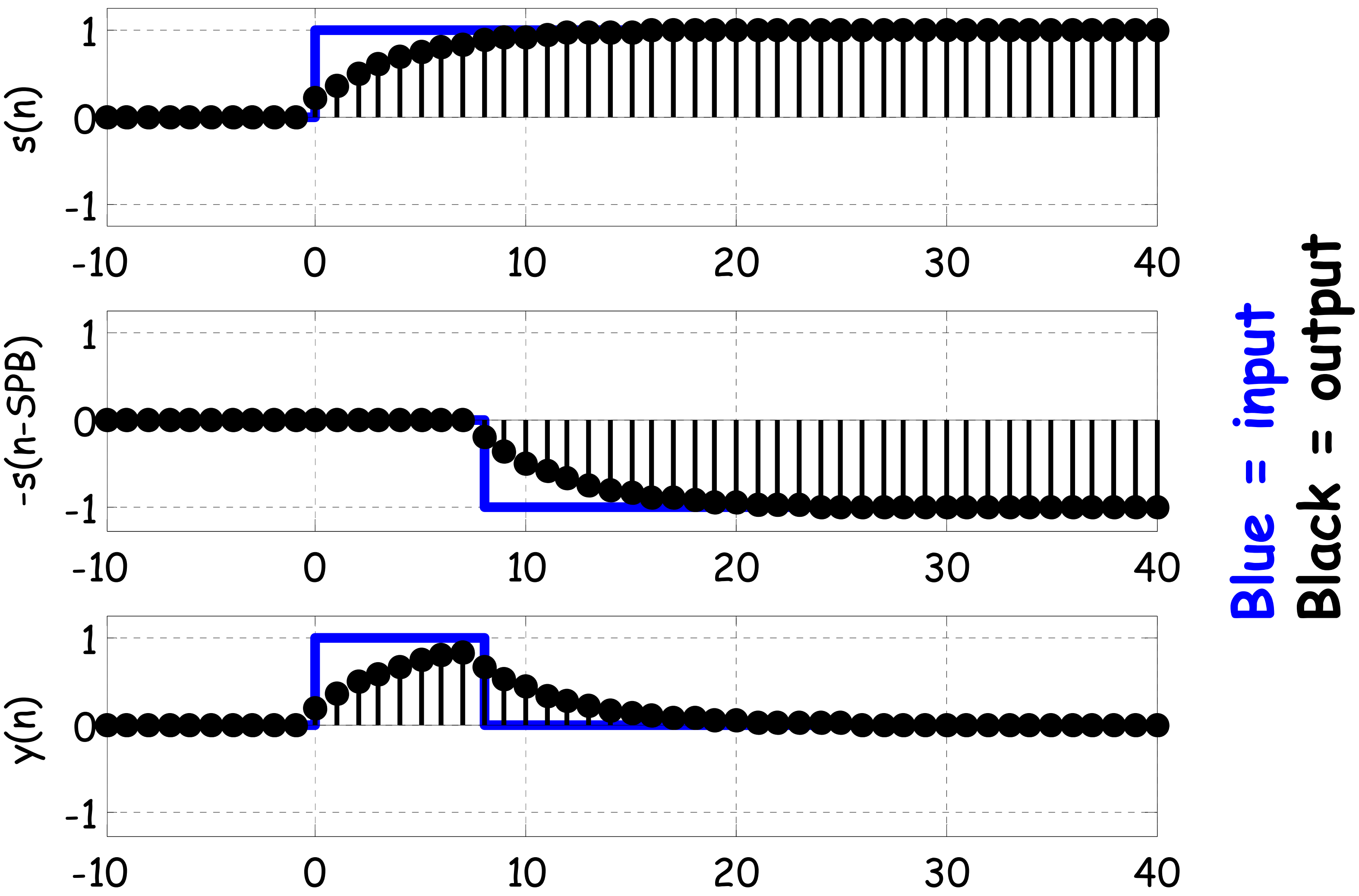
s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to single bit (SPB = 16)



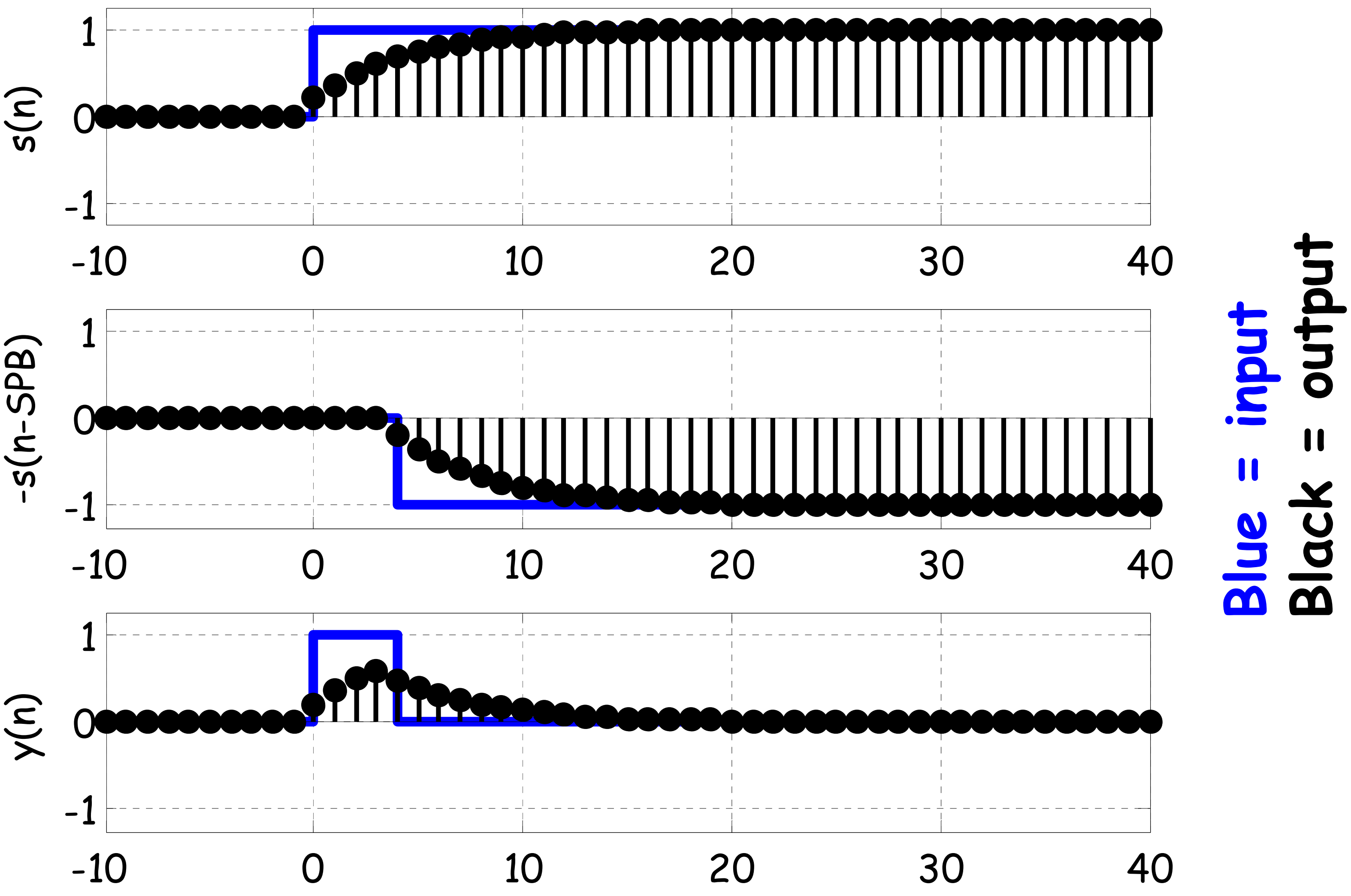
s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to single bit (SPB = 8)



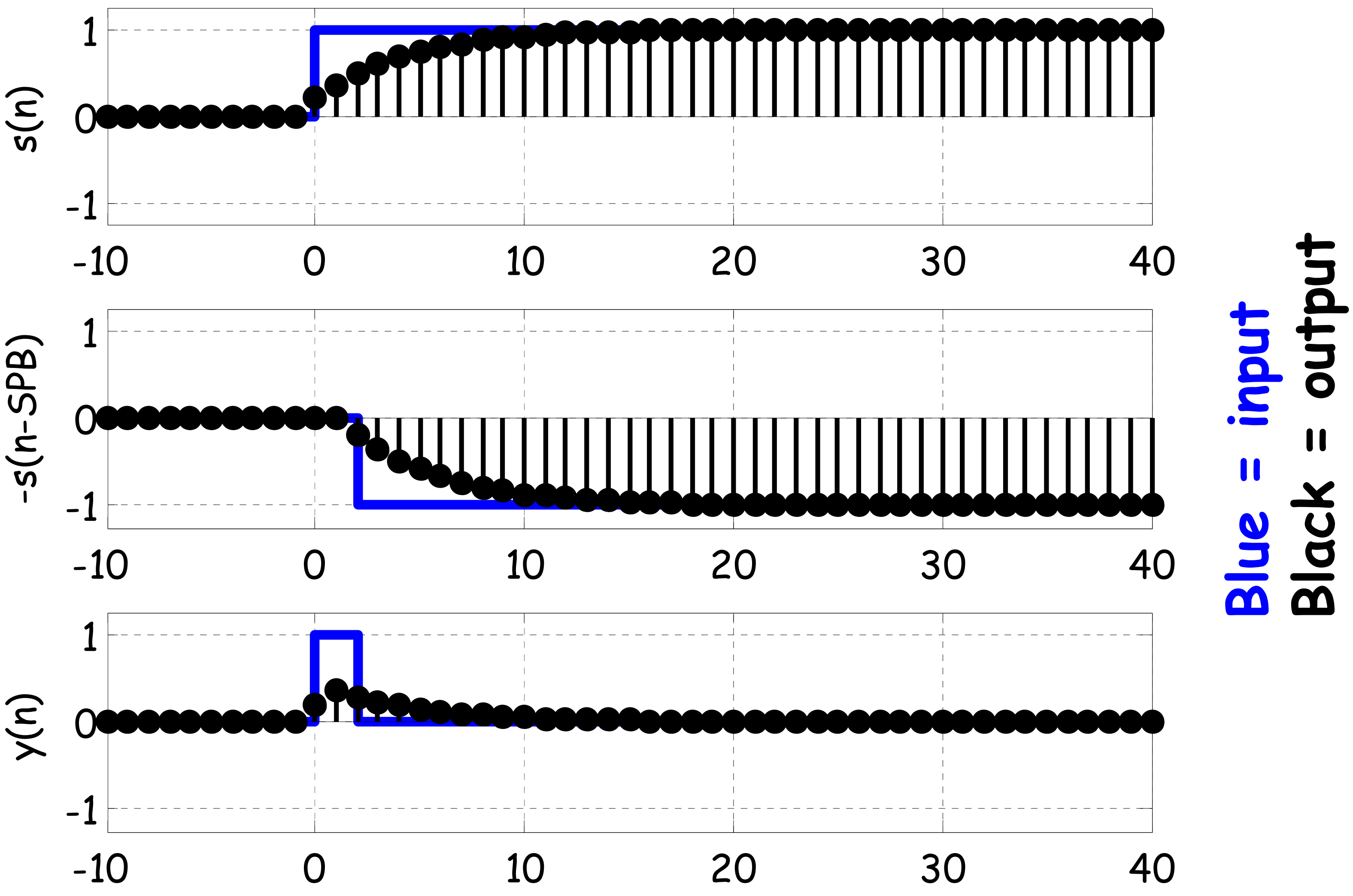
s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to single bit (SPB = 4)



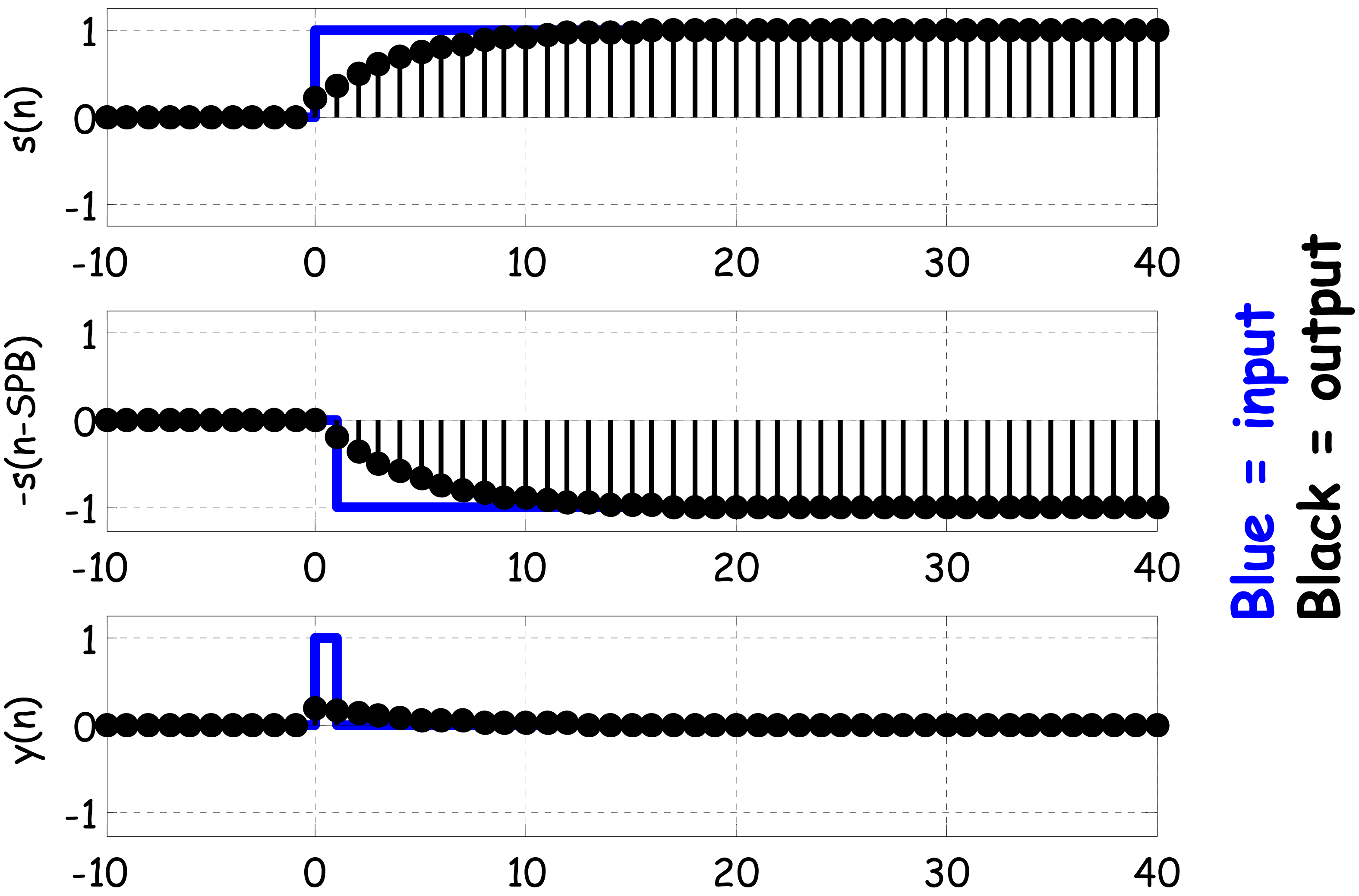
s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to single bit (SPB = 2)



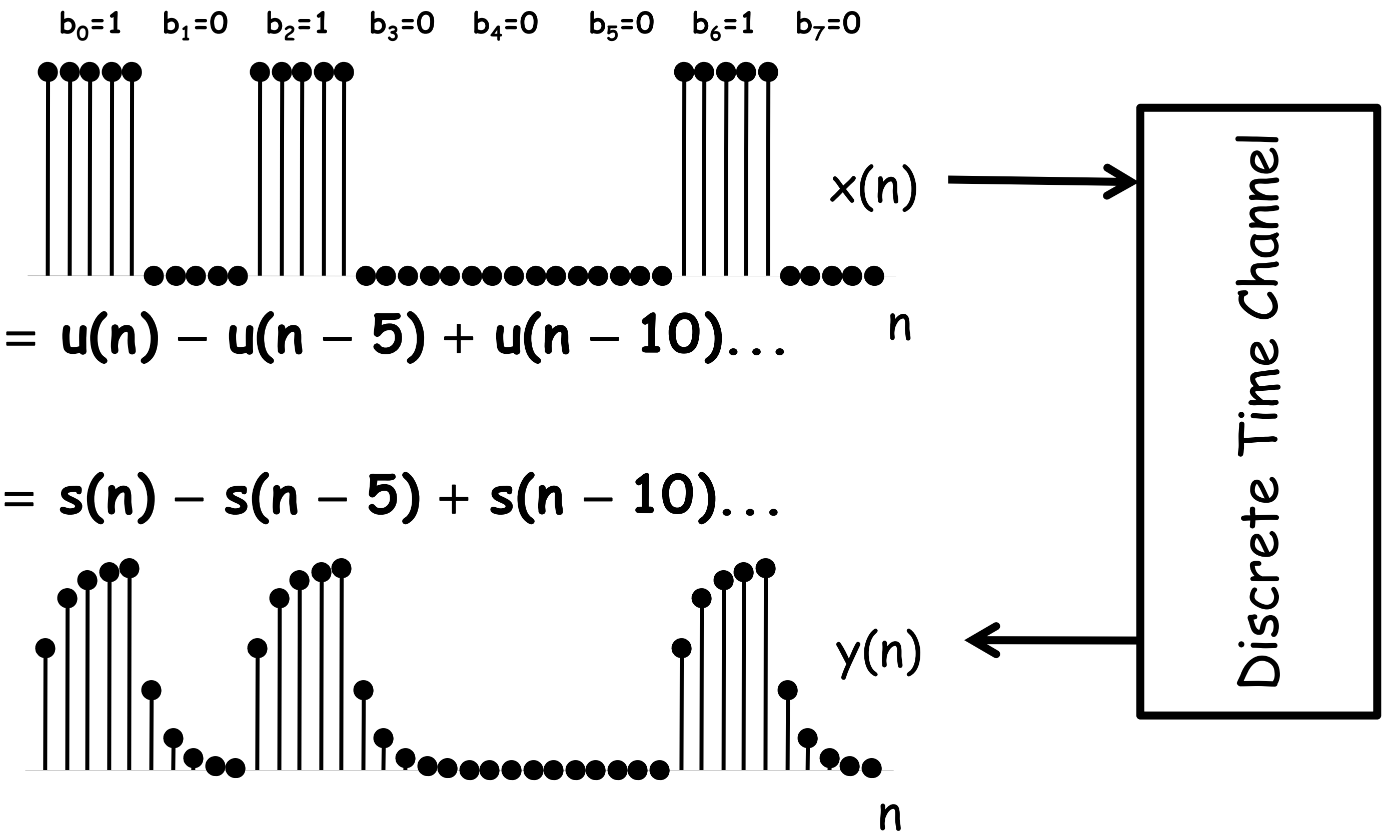
s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to single bit (SPB = 1)



s(n) = Exponential approach, a = 0.8, k = 1, D = 0

Response to more general input



$$x(n) = u(n) - u(n-5) + u(n-10)\ldots$$

$$y(n) = s(n) - s(n-5) + s(n-10)\ldots$$

- **Important caveat (thing to be careful about)**
  - **If we add an offset or noise, then the channel is no longer LTI**
  - **However, we can still deal with this by considering the output to be the sum of the output of a LTI system plus the offset/noise.**