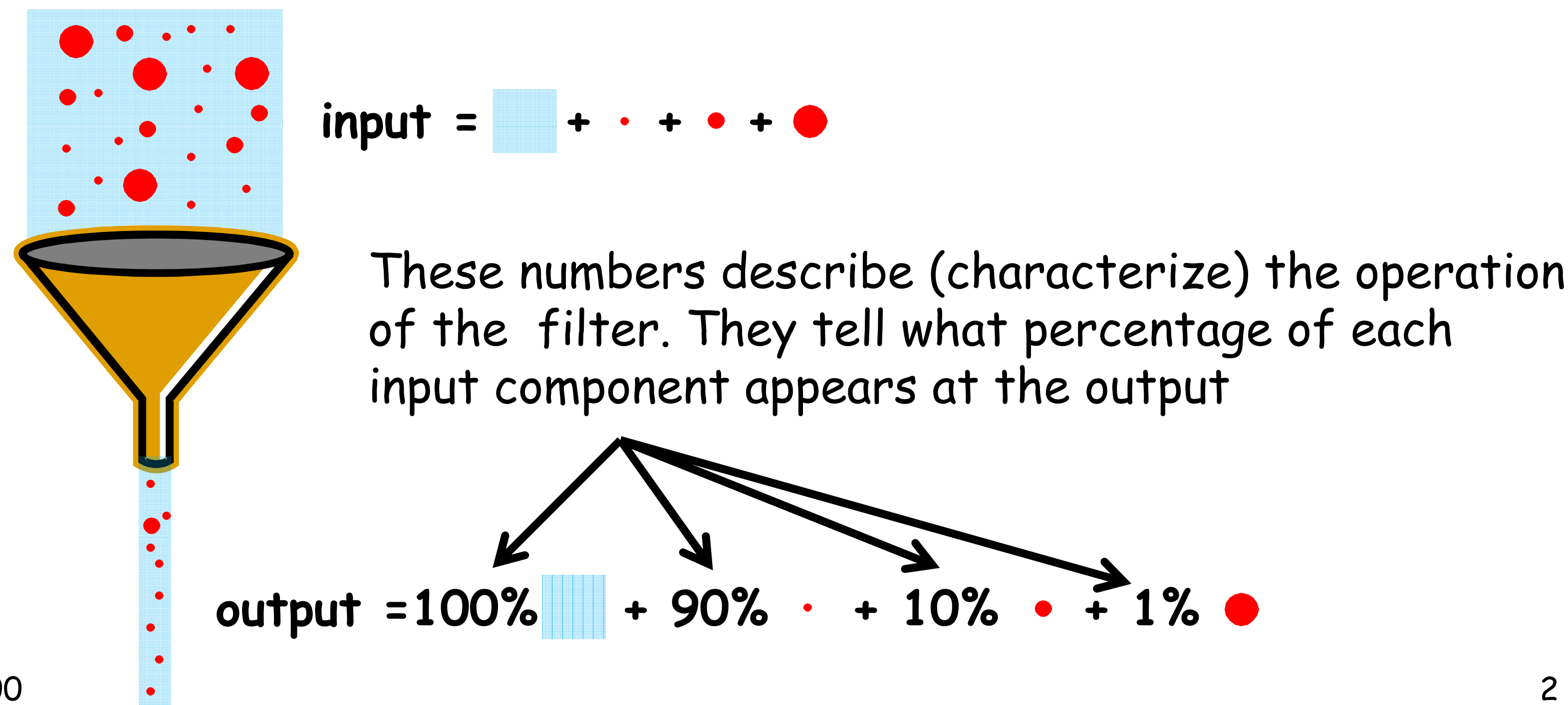
ELEC1200: A System View of Communications: from Signals to Packets
Lecture 13

- **Motivation: Filters**
- **Review: Fourier series expansion**
- **Frequency Response**
  - The effect of an LTI system on sinusoids
    - Scale in amplitude
    - Shift in phase
  - Filter types
- **Summary**

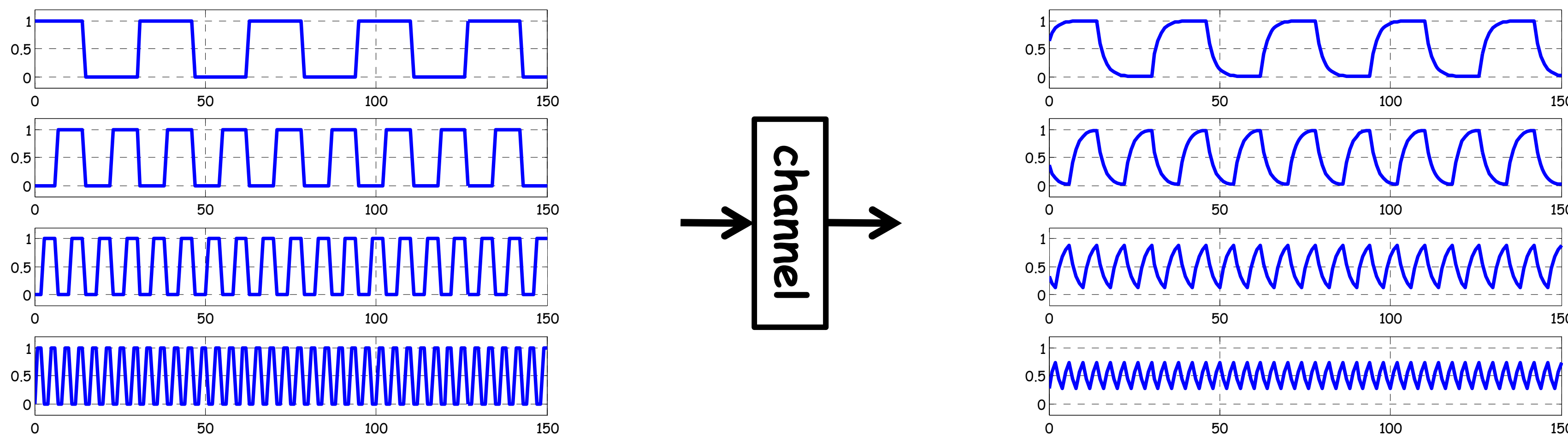
Motivation: Filters

- A filter is something that allows only certain parts (components) of the input to pass through to the output.
- For example, a water filter might let all small molecules (e.g. water) through, but block most large particles.



Channels as Filters

- A communication channel can be viewed as a filter.
- Some parts of the signal (e.g. the long flat parts) pass through, but others (e.g. the fast changes) do not

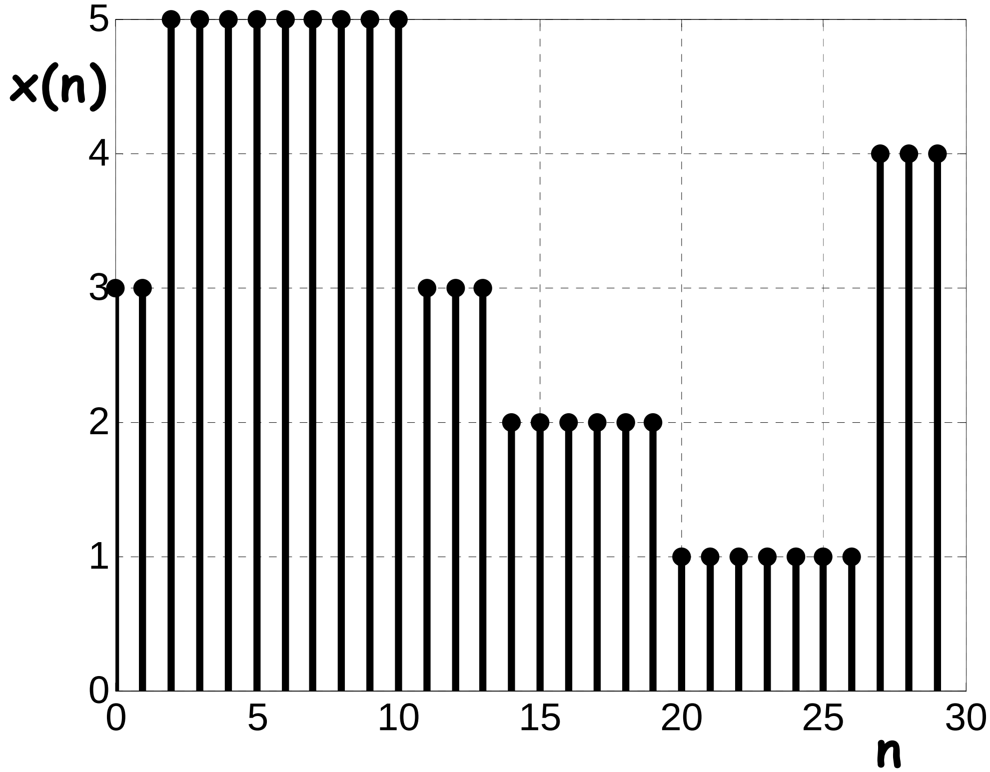


- Questions we answer in this lecture:
  - What are the "components" of the input signal?
  - How do we describe how much of each component is passed from input to output

Signal Components

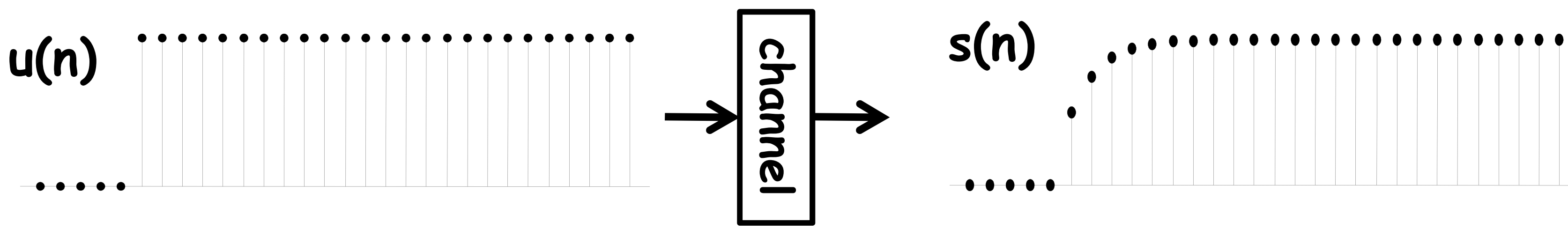
- We can decompose (break into parts) any signal by expressing it as a sum of "simpler" signals.
- For example, we have already seen that we can express any signal as the sum of scaled and delayed unit steps.

$$
\begin{aligned}
x(n) = {} & 3 \cdot u(n) \\
& + 2 \cdot u(n-2) \\
& - 2 \cdot u(n-11) \\
& - u(n-14) \\
& - u(n-20) \\
& + 3 \cdot u(n-27)
\end{aligned}
$$



Problem with Decomposing as Unit Steps

- If we treat unit steps as our basic components, then the components of the output don't "look like" the components of the input.



- We wish to find a set of components such that
  - Any signal can be expressed as the sum of scaled and delayed versions of these components
  - The components still "look the same" after passing through the channel
- It turns out that cosine/sine waves can do this!

Review: Fourier Series

- In this lecture, we show how to describe an LTI system as a filter.
- To do this, we express the input and output signals using the Fourier Series expansion studied in the last lecture:

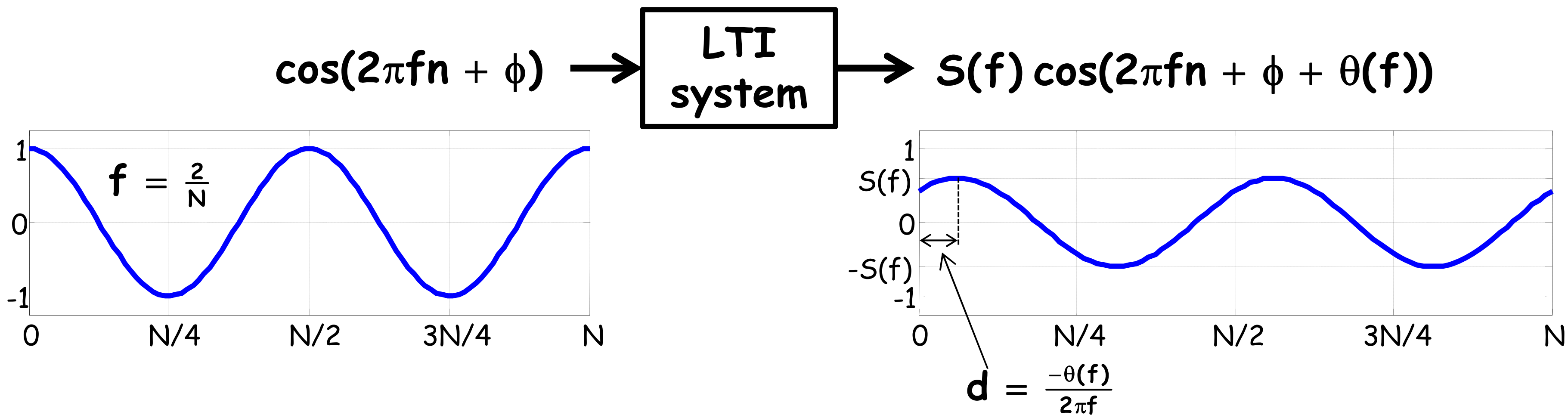
$$x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k) \qquad f_k = \frac{k}{N}$$

$$= \sum_{k=0}^{N/2} A_k \cos(2\pi f_k n + \phi_k)$$

- Each signal is a combination of components which are cosines that are scaled in magnitude and shifted in phase.

Effect of LTI systems on Cosines

- The response of any LTI system to an input cosine wave is a cosine wave with the same frequency f, but scaled in amplitude by S(f) and shifted in phase by θ(f).



- The scaling and shift are frequency dependent
  - S(f) is known as the <u>amplitude response</u> of the filter
  - θ(f) is known as the <u>phase response</u>
  - Together, they are referred to as the <u>frequency response</u>.

Computing the Output of an LTI system

- Step 1: Express the input as the sum of cosines
- Step 2: Scale and shift each cosine
- Step 3: By linearity, add the results together

step 1

$$x(n) = A_1 \cos(2\pi f_1 n + \phi_1) + A_2 \cos(2\pi f_2 n + \phi_2) + \ldots$$

LTI system

step 2

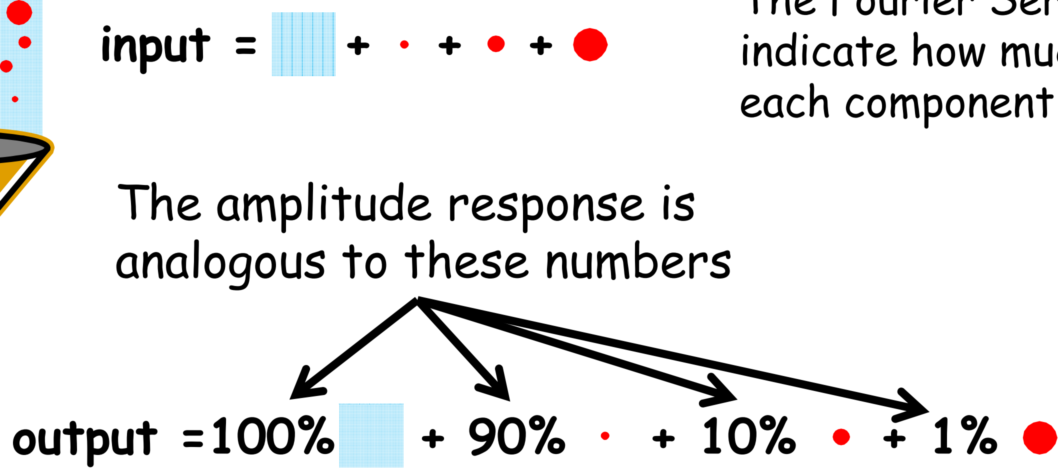
step 2

$$y(n) = S(f_1)A_1 \cos(2\pi f_1 n + \phi_1 + \theta(f_1)) + S(f_2)A_2 \cos(2\pi f_2 n + \phi_2 + \theta(f_2)) + \ldots$$

step 3

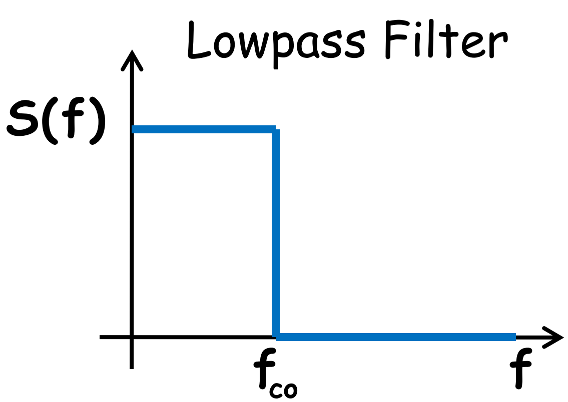
- This gives the same answer as when we express the input as a sum of unit steps and add the responses to each step.

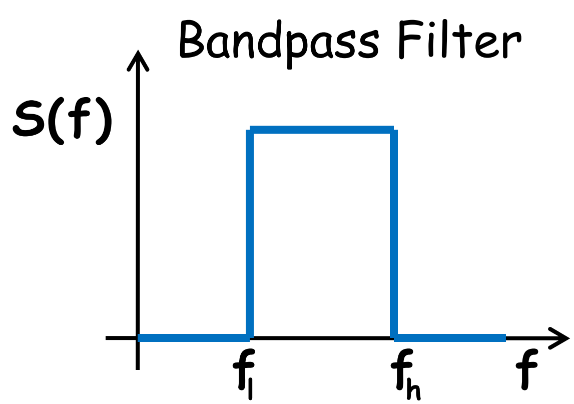
Amplitude Response

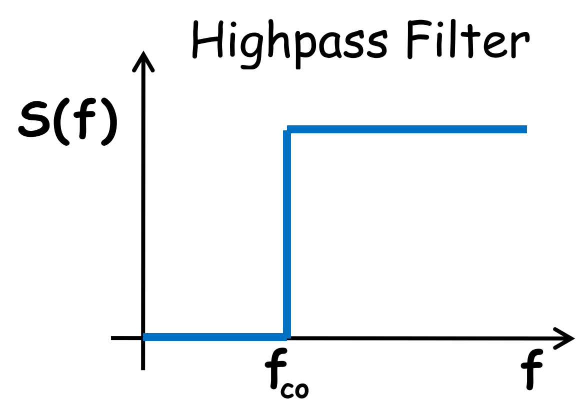


The Fourier Series coefficients indicate how much of each component we have

- **The plot of amplitude response versus f tells us what the filter does.**

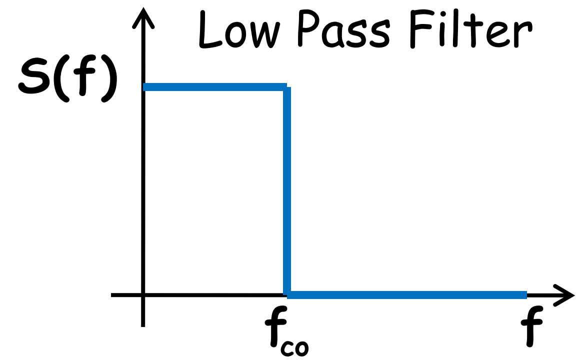


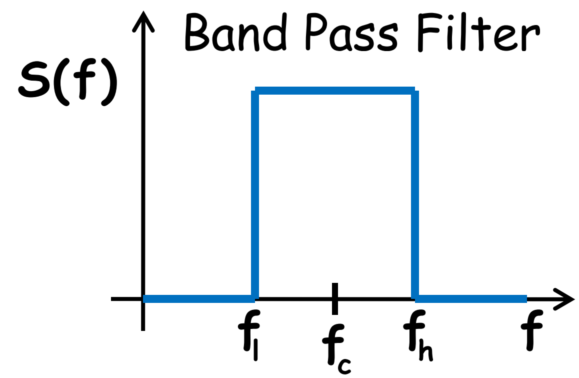


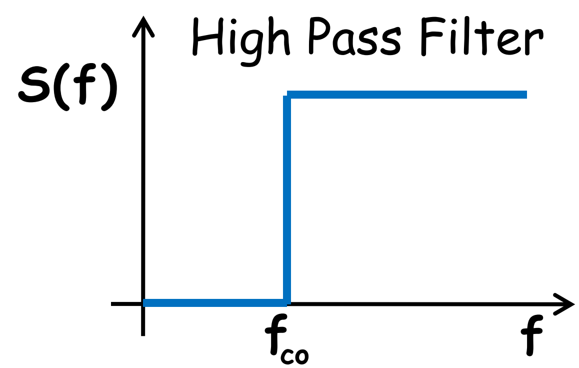


Types of Filters

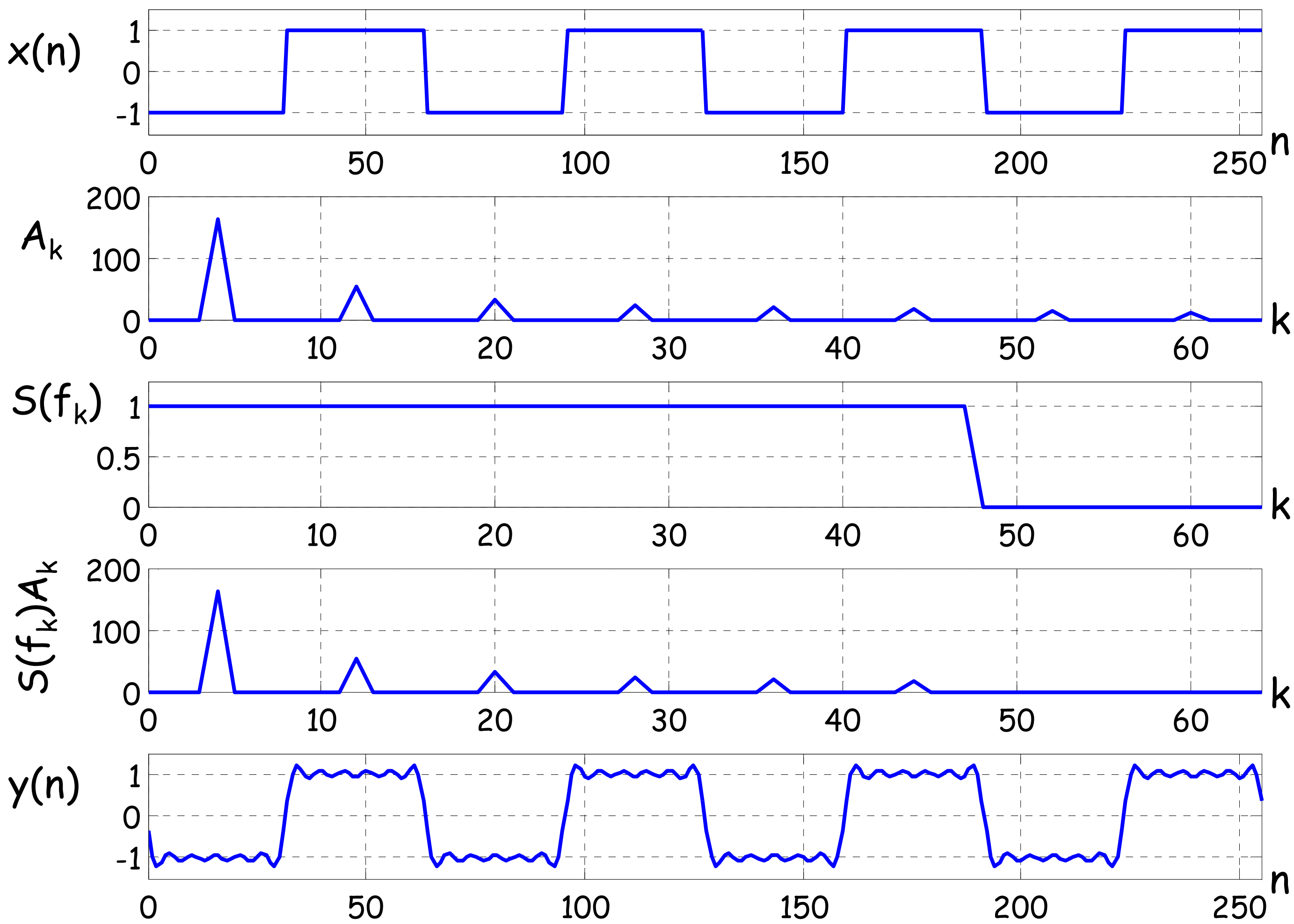
- Low Pass filter
  - Passes all frequencies below f_{co}
    - f_{co} = cutoff frequency
  - Blocks frequencies above f_{co}
  - Emphasizes "bass"
- Band Pass filter
  - Blocks all frequencies below f_l
    - f_l = low frequency cutoff
  - Blocks frequencies above f_h
    - f_h = high frequency cutoff
  - Passes frequencies around f_c
    - f_c = center frequency
  - Emphasizes "midrange"
- High Pass filter
  - Blocks all frequencies below f_{co}
    - f_{co} = cutoff frequencies
  - Passes frequencies above f_{co}
  - Emphasizes "treble"

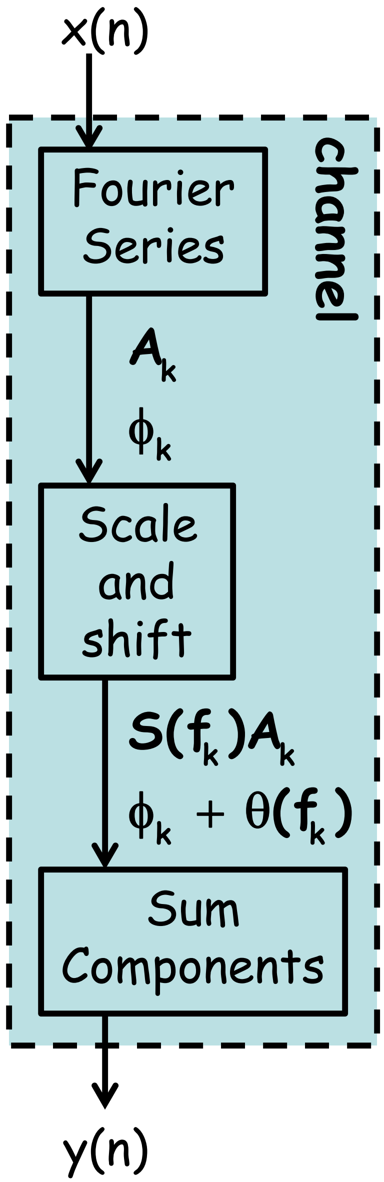




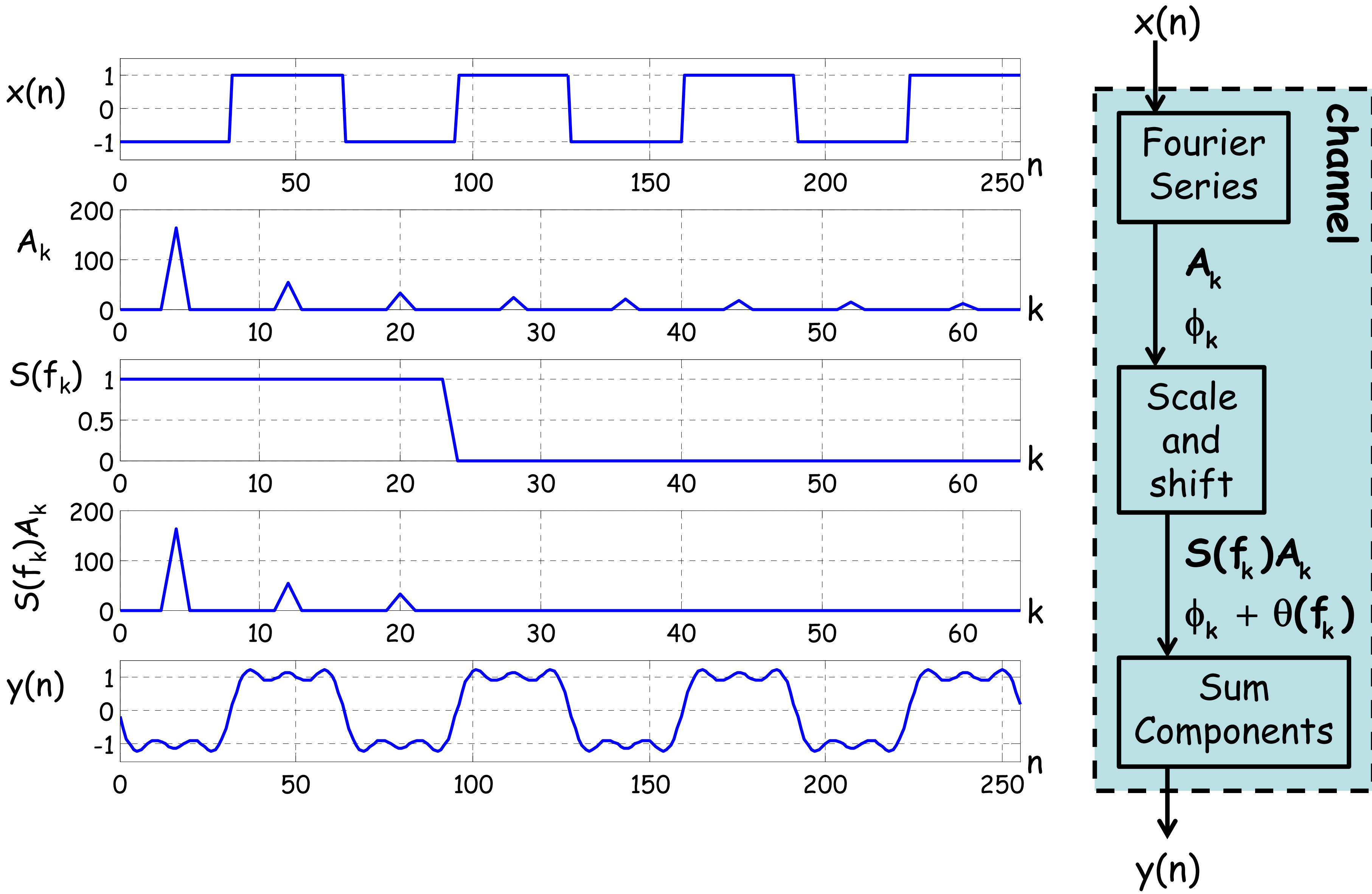


Example: Low Pass Filter f_{co} = 48/256

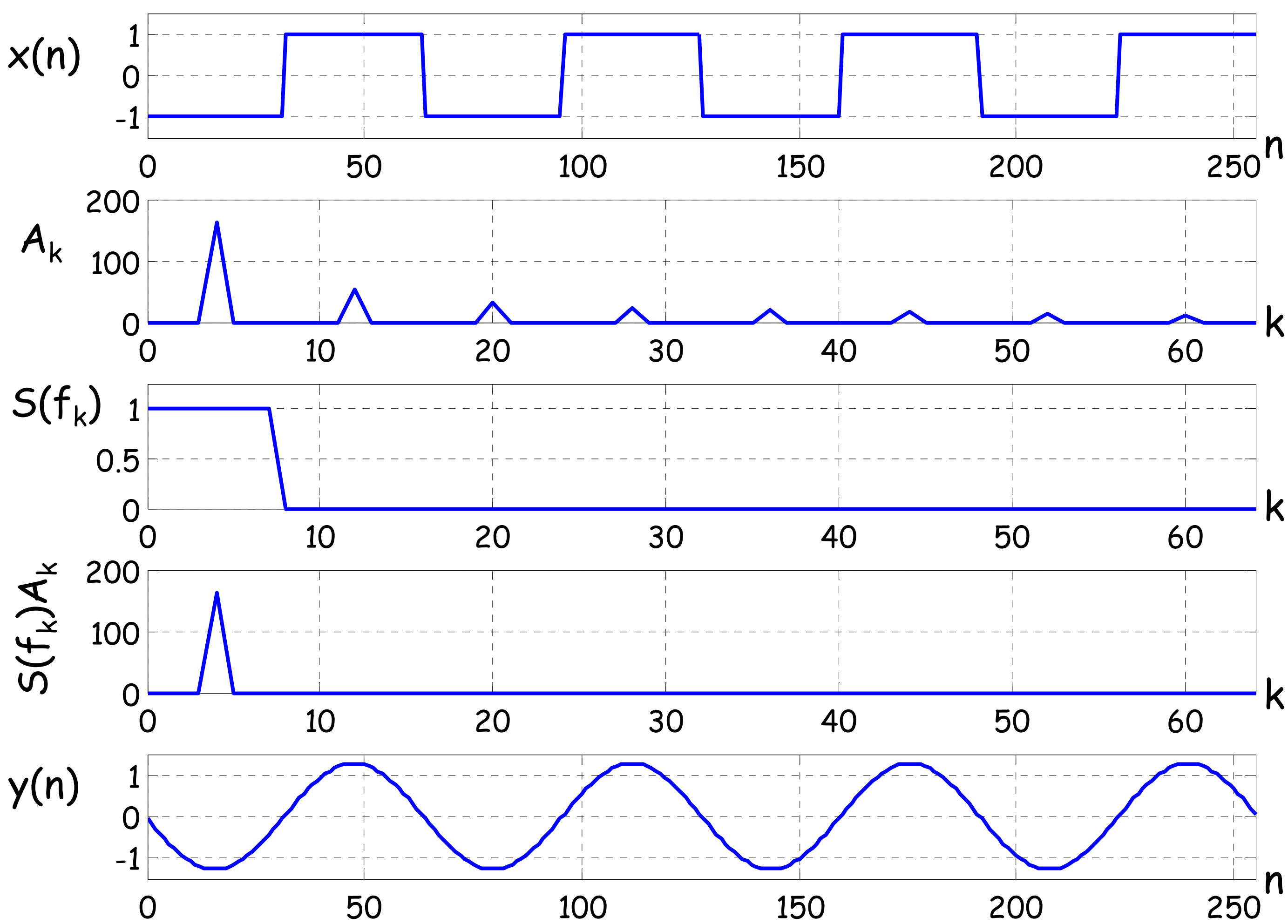


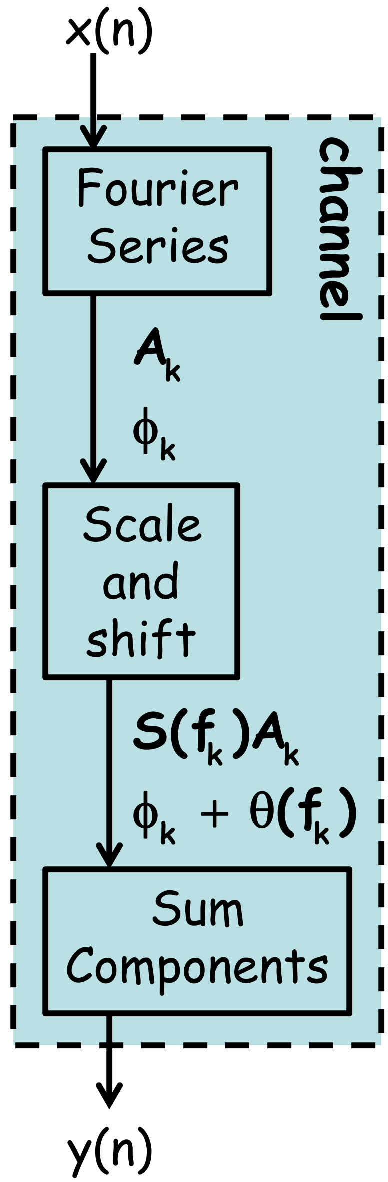


Example: Low Pass Filter f_{co} = 24/256

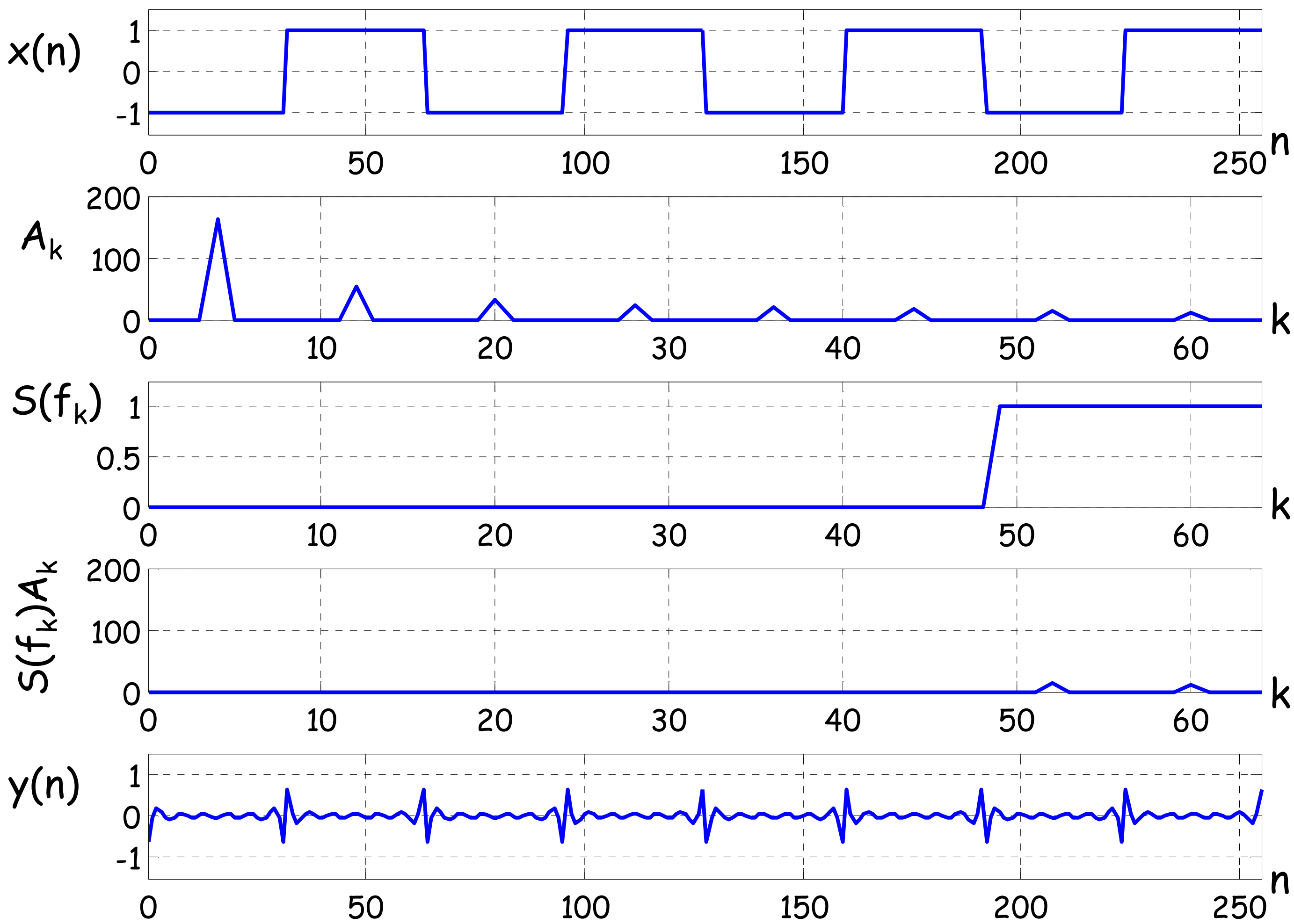


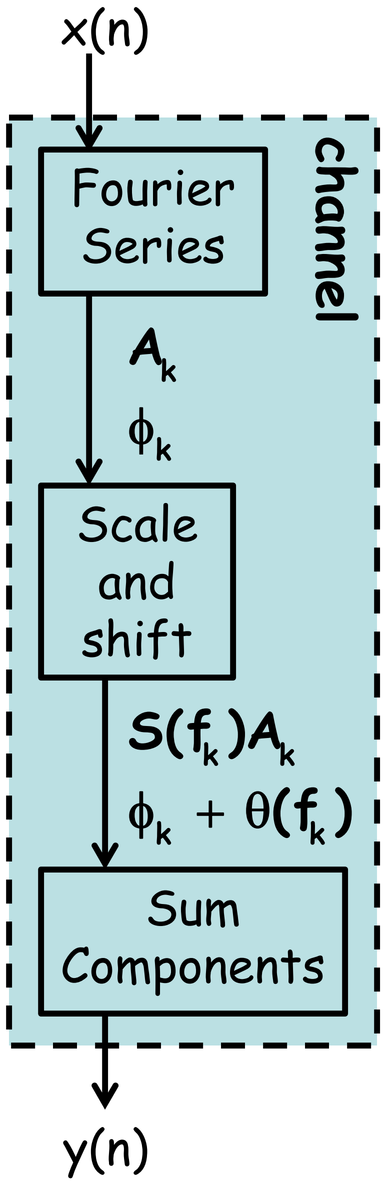
Example: Low Pass Filter f_{co} = 8/256



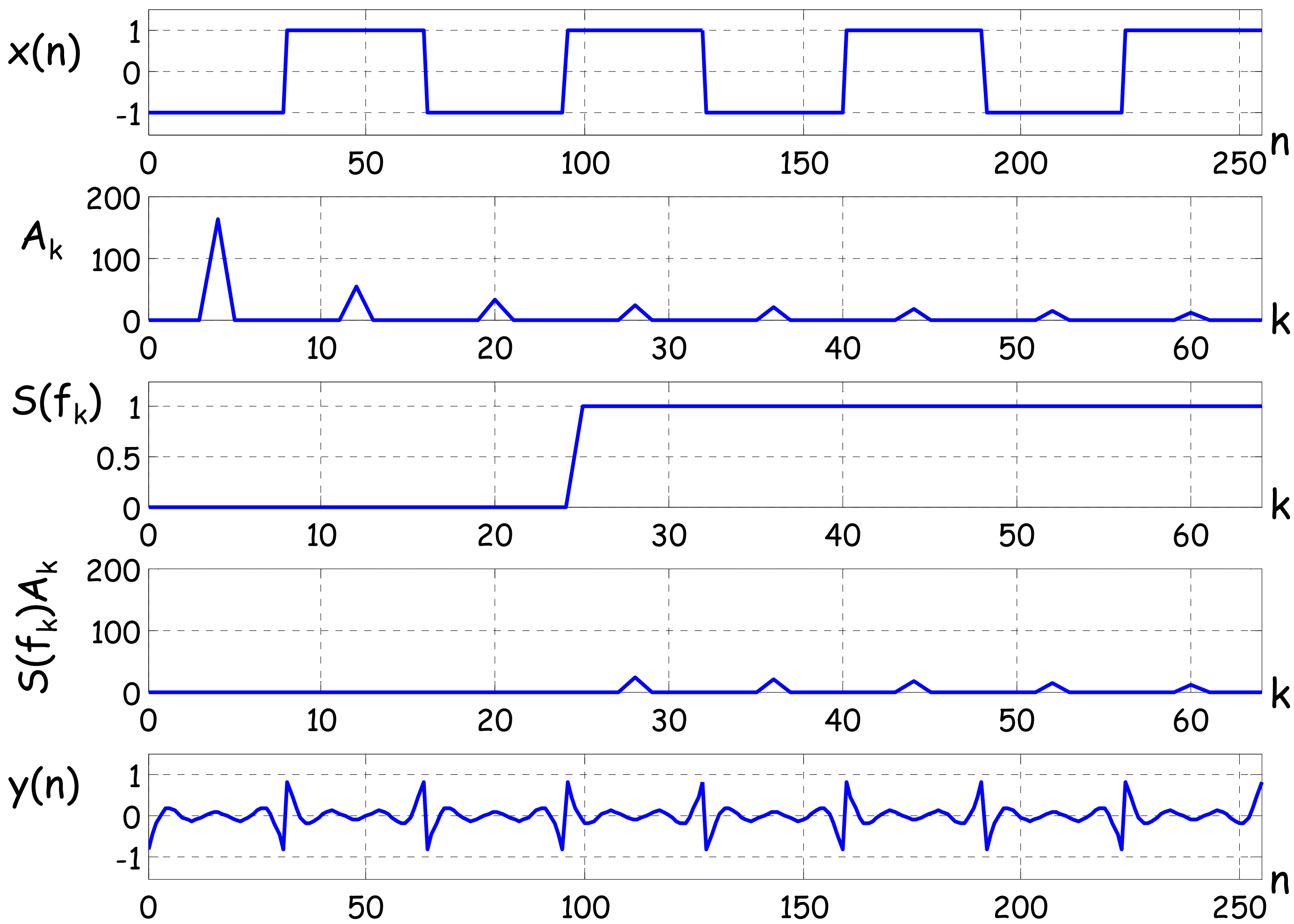


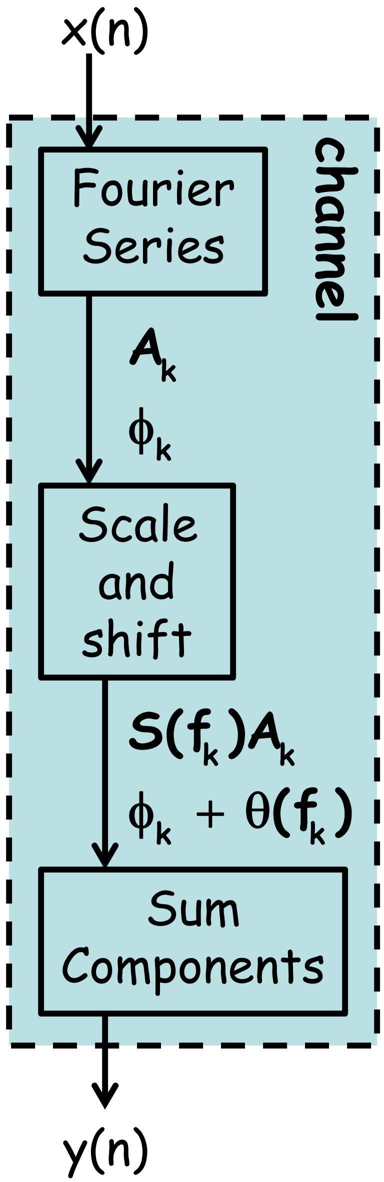
Example: High Pass Filter f_{co} = 48/256



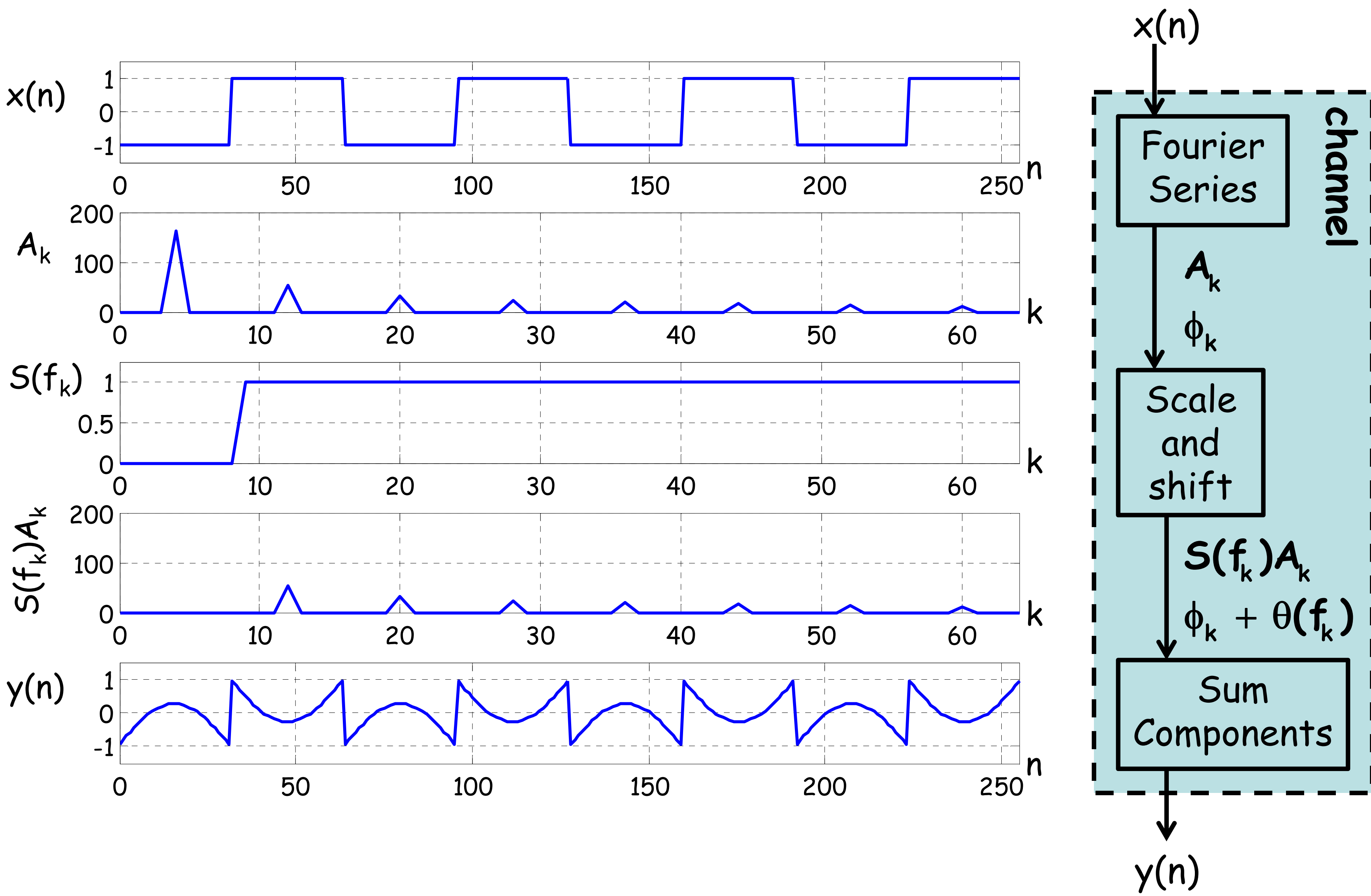


Example: High Pass Filter f_{co} = 24/256

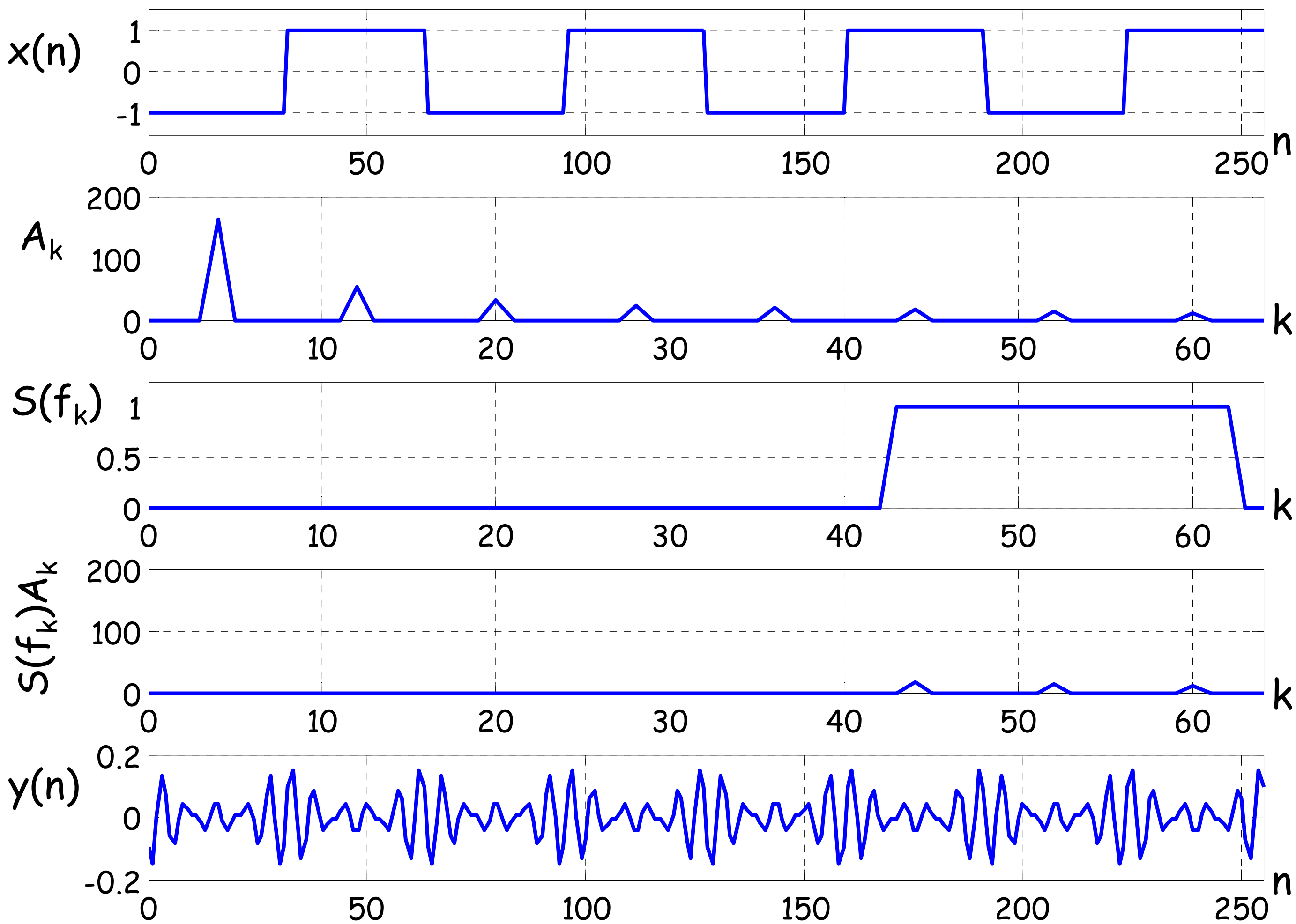


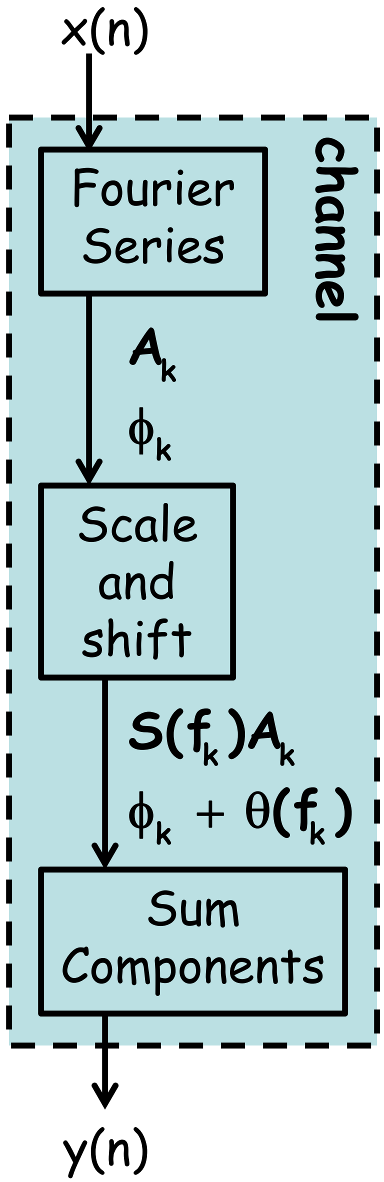


Example: High Pass Filter f_{co} = 8/256

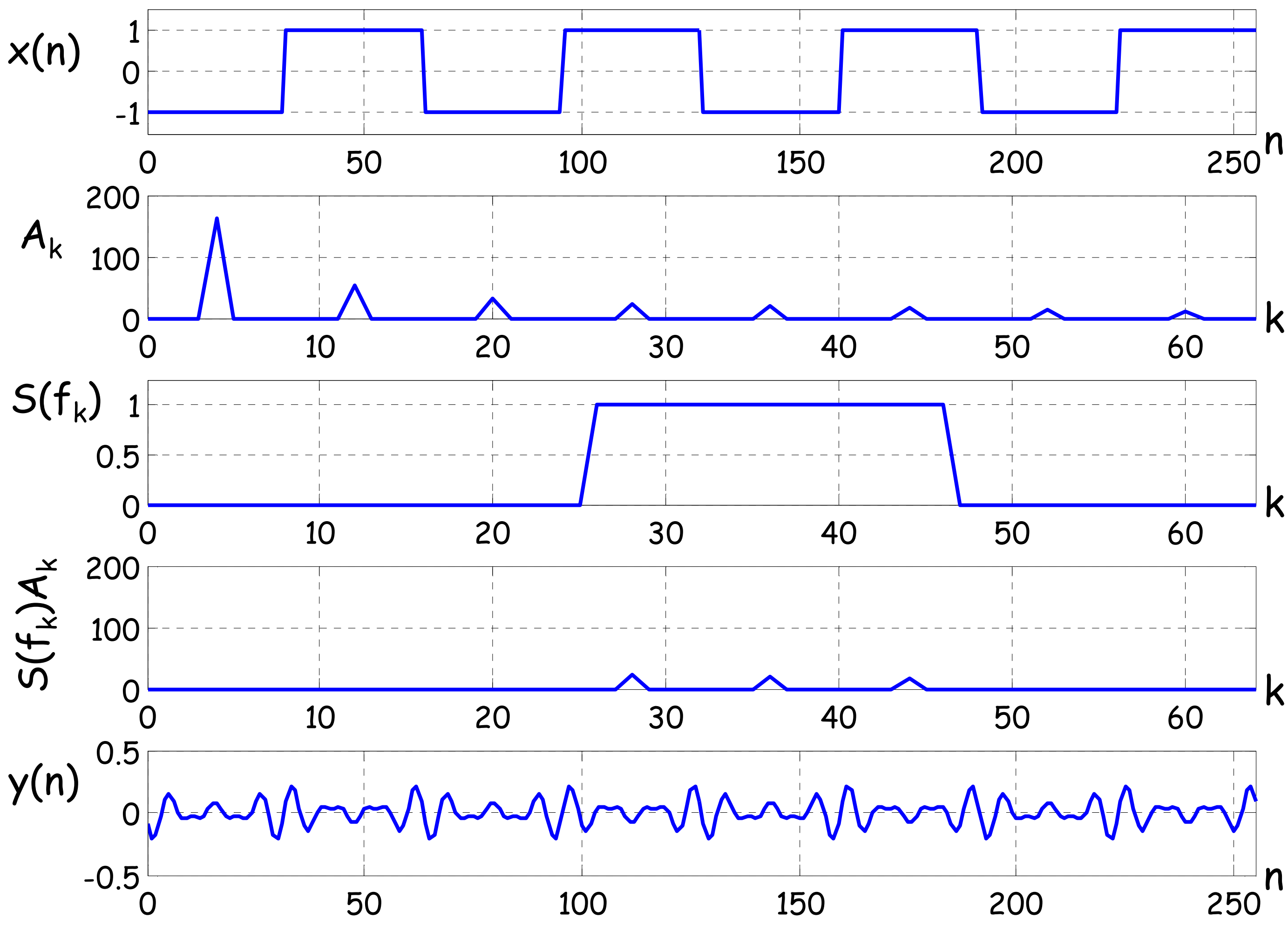


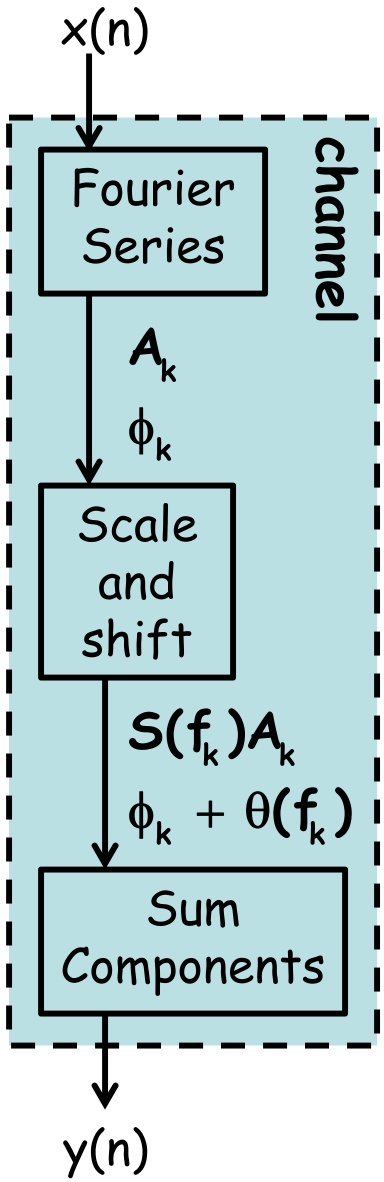
Example: Band Pass Filter f_c = 52/256



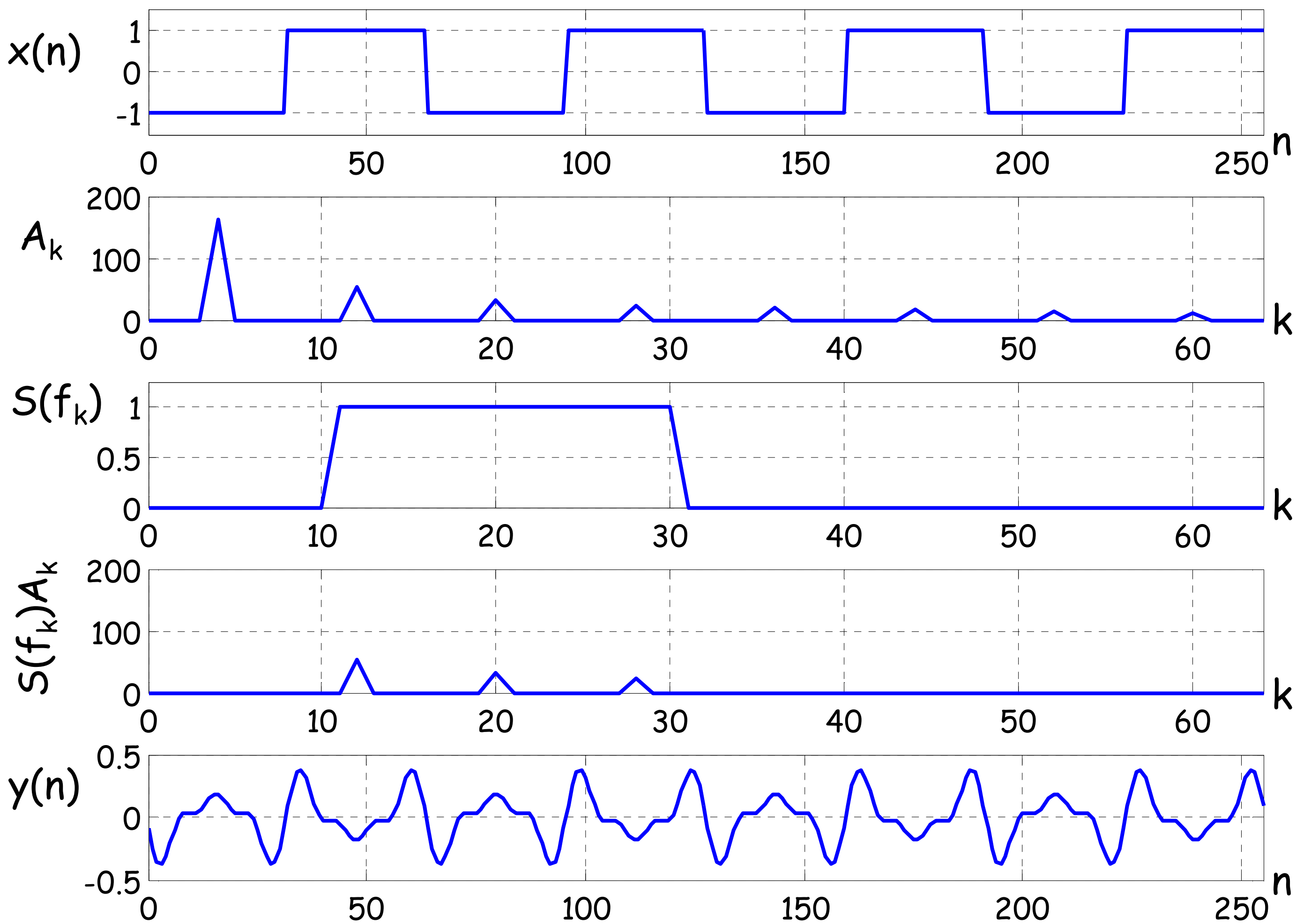


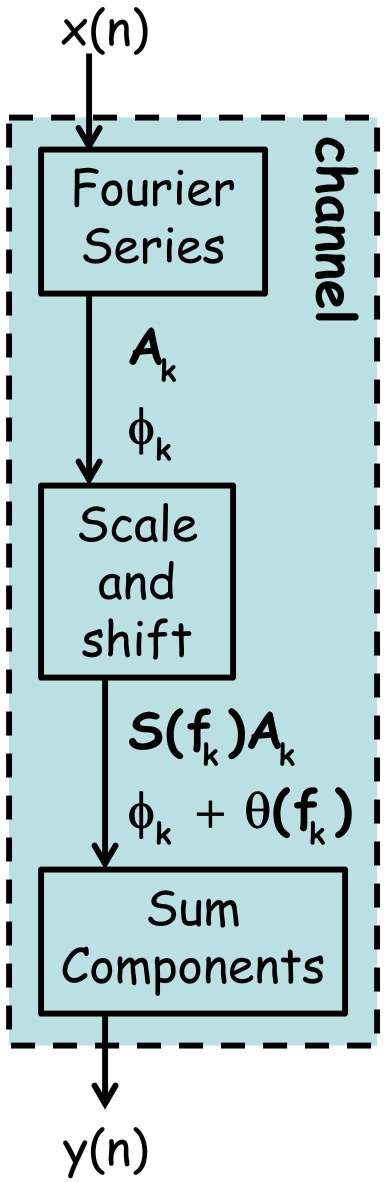
Example: Band Pass Filter $f_c = 36/256$





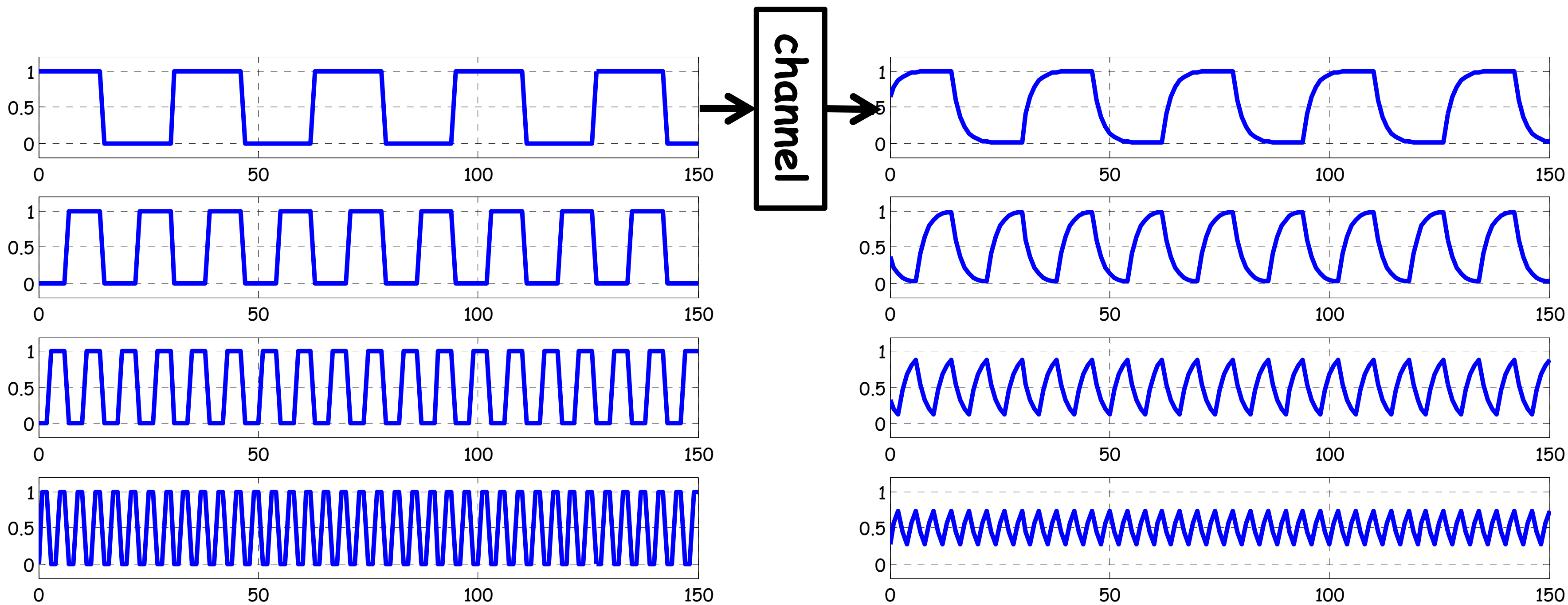
Example: Band Pass Filter f_c = 20/256





The IR Channel

- The long flat parts (low frequencies) pass through the IR channel in the lab, but the sharp transitions (high frequencies) do not.
- Thus, we expect the IR channel to have a "low pass" characteristic.
- We will measure this in lab.

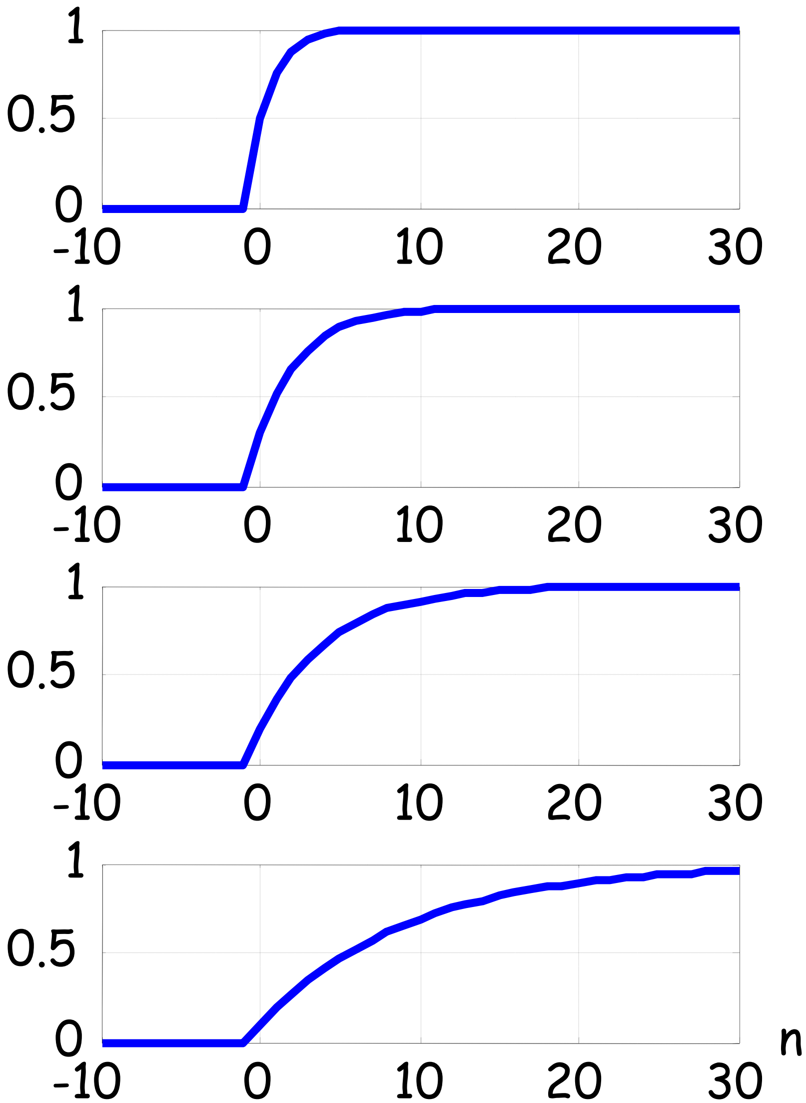


Step Response vs. Frequency Response

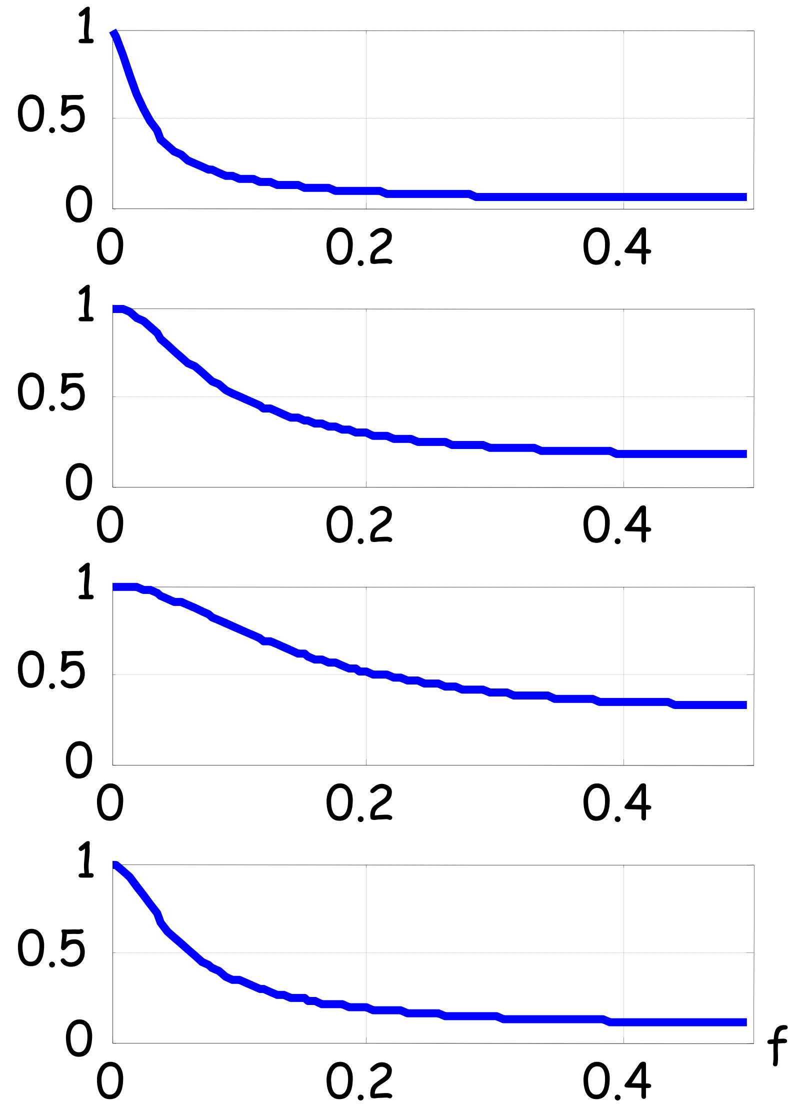
- The step response and the frequency response are equivalent
  - Both provide enough information to compute the output of a channel in response to any input.
  - One can be computed from the other
- Like the signal and its Fourier Series expansion, they are just different ways of looking at the same information
  - Step response shows how the channel output changes in time.
    - Useful for predicting the eye diagram and responses to bit waveforms.
  - Frequency response shows how the channel output responds to different frequencies.
    - Useful to understand the response to more complex signals.

Which step goes with which frequency?

Step Response



Amplitude Response



Summary

- **Using linearity, we can compute the response of an LTI system to any input by**
  - **Expressing the signal as a sum of cosines of different frequencies (Fourier Series Expansion)**
  - **Scaling each cosine by the amplitude response S(f) and shifting it by the phase response q(f)**
  - **Summing the resulting cosines to obtain the output**
- **This is known as working in the frequency domain.**
- **This gives the same answer as would be obtained by looking at the step response.**
  - **You are free to choose which to use, based on which is easier.**
- **Filters (LTI systems) are classified according to the shape of their amplitude response.**

Appendix: Frequency Response

- The scaling $S(f)$ and phase shift $\phi(f)$ due to passing the cosine through an LTI system can be computed from a complex valued function $H(f)$, called the <u>frequency response</u> of the system

$$\cos(2\pi f_k n) \longrightarrow \boxed{\text{LTI system } H(f)} \longrightarrow S(f_k)\cos(2\pi f_k n + \theta(f_k))$$

frequency response magnitude (more important)

$$S(f_k) = \left|H(f_k)\right|$$

$$\phi(f_k) = \angle H(f_k)$$

frequency response phase (less important)

$$H(f_k) = S(f_k)e^{j\theta(f_k)}$$
$$= S(f_k)\left(\cos(\theta(f_k)) + j\sin(\theta(f_k))\right)$$

$$j = \sqrt{-1}$$