ELEC1200: A System View of Communications: from Signals to Packets
Lecture 11

- Review of (8,4) code
- (9,4,4) code
- Burst Error Correction
  - Interleaving
  - Framing
  - Bit Stuffing

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Review: Error Correcting Coding

1. Take an input message stream (here 12 bits).

2. Break the message stream into k-bit blocks (here 3 blocks of k = 4 bits).

3. Add (n-k) parity bits to form n-bit codeword (here n = 8).

4. Transmit data through noisy channel and receive codewords with some errors

5. Perform error correction

6. Extract the k message bits from each corrected codeword.

011011101101

0110
1110
1101

01101111
11100101
11010110

channel

01100111
11110101
11000110

0110 1110 1101

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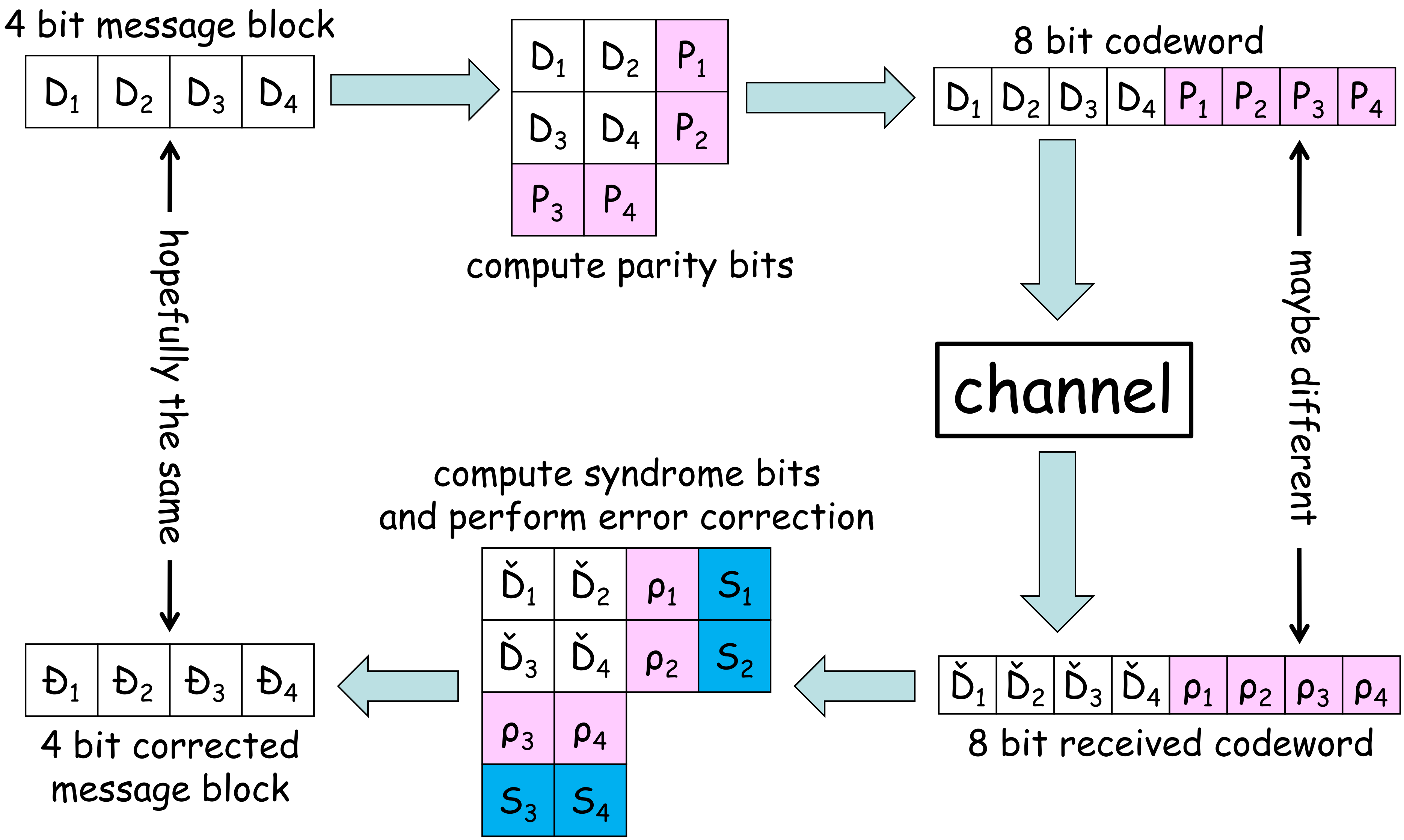
Review: (8,4) code



Performing Error Correction

- Take the code word, and rearrange it

D_1	D_2	D_3	D_4	P_1	P_2	P_3	P_4

D_1	D_2	P_1
D_3	D_4	P_2
P_3	P_4	

- Compute the syndrome bit required for each row/column to be even parity

D_1	D_2	P_1	S_1
D_3	D_4	P_2	S_2
P_3	P_4		
S_3	S_4		

- Check the syndrome bits.
  - If all are 0, there is no error
  - If only syndrome bits for column i and row j are 1 (e.g. S_1 and S_3), there is a bit error in the data bit at i,j (e.g. D_1)
  - If only one syndrome bit is 1 (e.g. S_2), there is an error in the corresponding parity bit (e.g. P_2).

(n,k,d) Block Codes

- **These are the same as (n,k) codes, except that we indicate the minimum Hamming distance, d, between valid code words**
  - **For example, the (8,4) code studied before is an (8,4,3) code.**
- **The minimum Hamming distance determines how many bit errors we can detect or correct.**
  - **We can <u>detect but not correct</u>, errors in at most d-1 bits. (If d=3, we can detect 1 or 2 bit errors.)**

OR

  - **We can <u>detect and correct</u> errors in at most (d-1)/2 bits. (If d=3, we can detect and correct only 1 bit errors)**
- **Note we must choose which one of the above options we want to do.**

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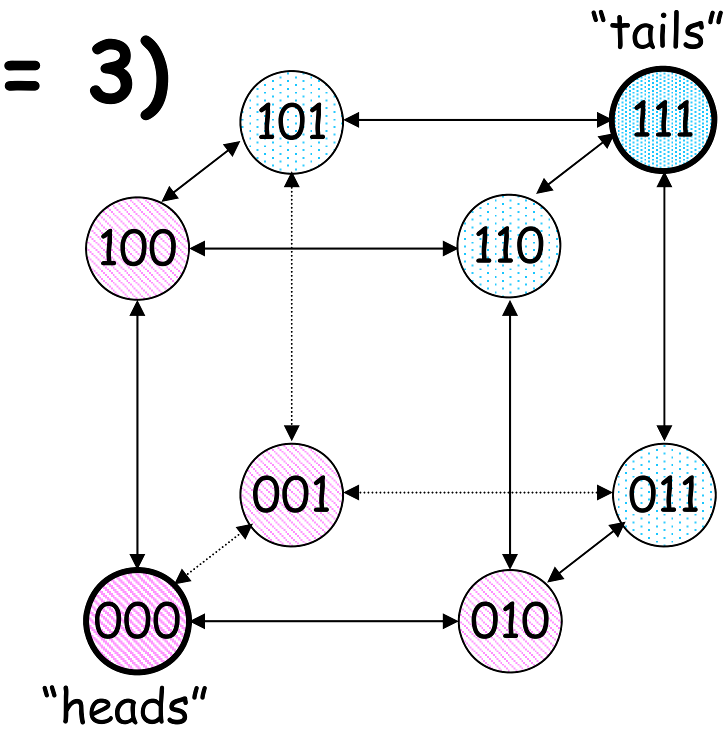
Example (d = 3)



- With a (n,k,3) code, we can EITHER
  - Detect 1-bit and 2-bit errors

    OR
  - Detect and correct 1-bit errors

  but not both!
- Suppose we observe the codeword 100 in the (3,1,3) code. EITHER
  - 000 was sent and a 1 bit error occurred

    OR
  - 111 was sent and a 2 bit error occurred.
- If we choose to correct 100 to 000, we are assuming that 2 bit errors never occur.

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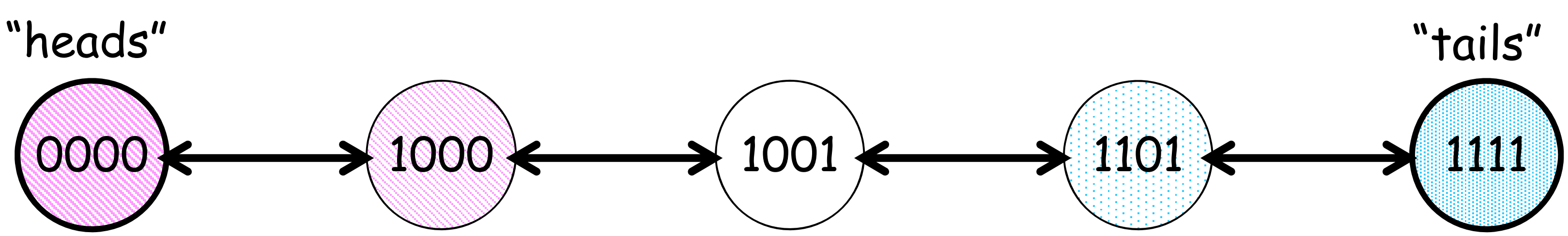
Example (d = 4)

- With a (n,k,4) code, we can EITHER
  - Detect 1-, 2-, and 3-bit errors

    OR
  - Detect and correct 1-bit errors and detect 2- bit errors
- For example, consider the (4,1,4) code:
  - If we observe 1001, we cannot determine whether 0000 or 1111 was transmitted.
  - If we observe 1000, then either a 1 bit or a 3 bit error occurred. If we correct, we assume the 3 bit error did not occur.



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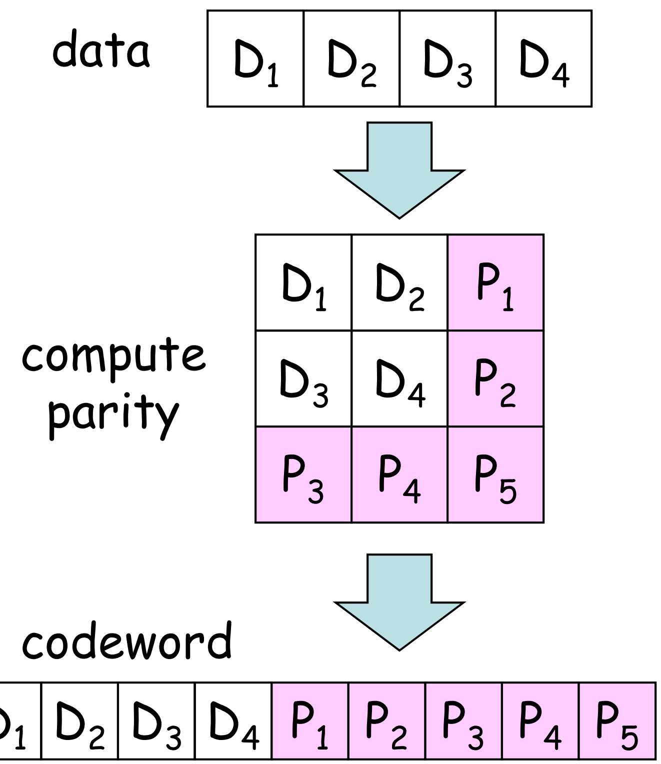
A (9,4,4) code

- We can increase the minimum Hamming distance in the (8,4,3) code to 4 by adding an overall parity bit P5, which is chosen so that the codeword always has even parity.
- Examples
  - Data Block: 1010
    - Compute Parity: 101
      101
      000
    - Codeword: 101011000
  - Data Block: 1110
    - Compute Parity: 110
      101
      011
    - Codeword: 111001011



Performing Error Correction

D_1	D_2	P_1	S_1
D_3	D_4	P_2	S_2
P_3	P_4	P_5	
S_3	S_4		S_5

- The (9,4,4) code allows us to
  - Detect and correct 1 Bit Errors

    AND
  - Detect 2 Bit Errors
- To do this
  - Compute the syndrome bits by checking each row/column and the entire codeword for even parity
  - Check the syndrome bits.
    - If all are 0, there is no error
    - If S_5=1, a 1 bit error occurred. Check the other syndrome bits to see which bit to correct (note that the parity bits may contain errors)
    - If S_5=0 but one or more of the other syndrome bit are nonzero, a 2 bit error occurred.
- Alternatively, we could detect 1,2 or 3 bit errors.

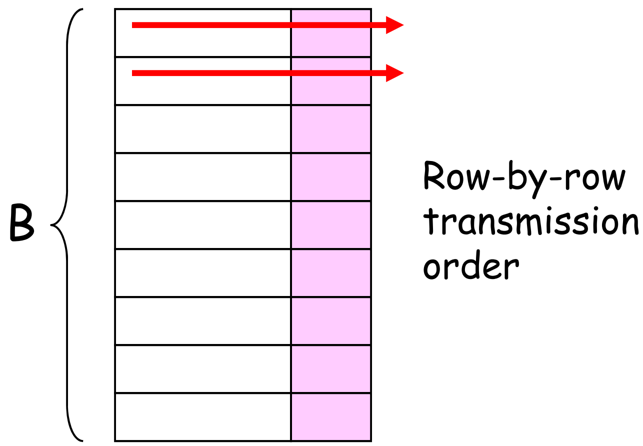
Examples of Error Correction

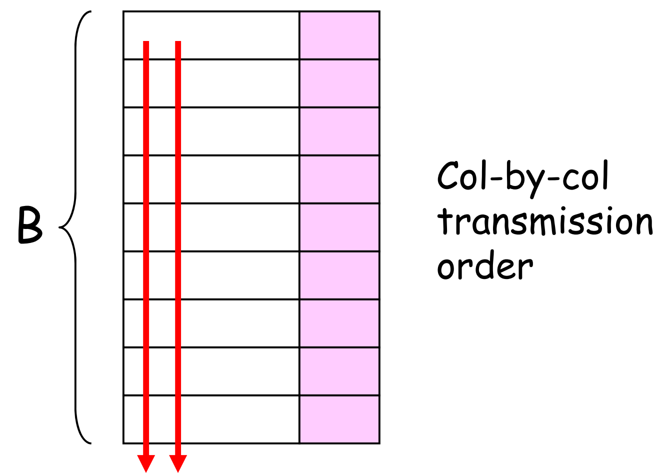
Received Codeword	Rearranged Codeword	Computed Syndrome	Corrected Data
011110101	0 1 1 1 1 0 1 0 1	0 1 1 0 1 1 0 0 1 0 1 0 0 0	0111 no errors
011010101	0 1 1 1 0 0 1 0 1	0 1 1 0 1 0 0 1 1 0 1 0 1 1	0111 D_4 incorrect
011111111	0 1 1 1 1 1 1 1 1	0 1 1 0 1 1 1 1 1 1 1 0 1 0	???? 2 bit error

Burst Errors

Correcting single-bit of few-bit errors is nice, but in many situations errors come in bursts many bits long (e.g., damage to storage media, burst of interference on wireless channel, …). How does single-bit or few-bit error correction help with that?

Well, can we think of a way to turn a B-bit error burst into B single-bit errors?





Problem: Bits from a particular codeword are transmitted sequentially, so a B-bit burst produces multi-bit errors.

Solution: interleave bits from B different codewords. Now a B-bit burst produces 1-bit errors in B different codewords.

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Framing

- **Looking at a received bit stream, how do we know where a block of interleaved codewords begins?**
- **Physical indication (transmitter turns on, beginning of disk sector, separate control channel)**
- **Place a unique bit pattern (frame sync sequence) in the bit stream to mark start of a block**
  - **Frame = sync pattern + interleaved code word block**
  - **Search for sync pattern in bit stream to find start of frame**
  - **Bit pattern can't appear elsewhere in frame (otherwise our search will get confused), so have to make sure no legal combination of codeword bits can accidentally generate the sync pattern**
  - **Sync pattern can't be protected by ECC, so errors may cause us to lose a frame every now and then, a problem that will need to be addressed at some higher level of the communication protocol.**

Bit Stuffing

- Each frame begins and ends with a special bit pattern called a sync pattern. For example, the sync pattern might consist of 6 consecutive ones surrounded by zeros

  [01111110]

- Inside each frame, whenever the sender encounters *five consecutive ones* in the data stream, it stuffs a 0 bit into the outgoing stream.
- When the receiver sees *five consecutive incoming ones* followed by a 0, it unstuffs (removes) the 0 bit.

Bit Stuffing Example

Input Stream

0110111111100111110111111111100000

Stuffed Stream

0110111110110011111001111101111100000

Stuffed bits

Unstuffed Stream

0110111111100111110111111111100000

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Summary: Channel Coding

1. Take an input message stream: 011011101101
2. Break the message stream into k-bit blocks (e.g. k = 4).
   0110
   1110
   1101
3. Add (n-k) parity bits to form n-bit codeword (e.g. n = 8)
   01101111
   11100101
   11010110
4. Interleave bits from B codewords (e.g., B = 3).
   011111110001100111101110
5. Bit-stuff the interleaved block
   01111011100011001111001110
6. Add the sync pattern (e.g. [0111110]).
   01111100111101110001100111100 1110

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Step 4: sync = 0111110

Summary: Error Correction

0111110011110111000110011110 01110

1. Receive bit stream (with errors)

0111110011110111001000011110 01110

2. Search for and remove sync pattern (0111110)

01111011100100001111001110

3. Destuff the frame

011111110010000111101110

4. De-interleave the bits to form B n-bit codewords (B=3, n=8)

01100111
11110101
11000110

5. Perform error correction

010	110	110
101	111	001
11	01	10

6. Extract the k=4 message bits from each corrected codeword.

0110 1110 1101

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Summary

- The minimum Hamming distance between code words determines how many bit errors in the codeword we can detect or correct.
- We can increase the minimum Hamming distance in the (8,4,3) code by adding an overall parity bit, resulting in a (9,4,4) code.
  - This adds the capability of detecting 2 bit errors.
- We can handle B-bit burst errors by
  - Interleaving B codewords
  - Adding a sync pattern so receiver can find the start of each block
  - Bit stuff the interleaved block so that sync is unique.

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