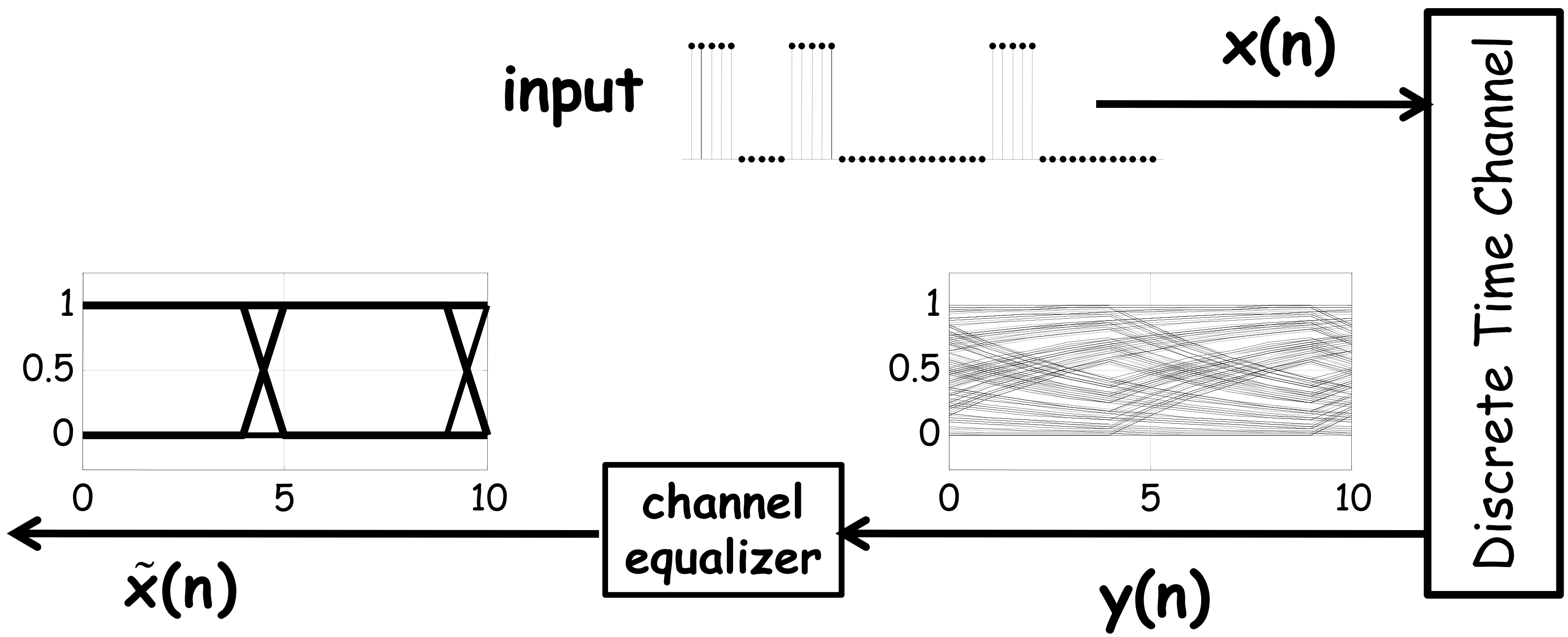
ELEC1200: A System View of Communications: from Signals to Packets
Lecture 7

- **Review:**
  - Motivation: Equalization
  - Modeling the IR channel using a recursive equation
- **Intuition for equalizer**
- **Derivation of equalizer**
- **Effect of equalization on the eye diagram**

Motivation: Equalization

- The channel introduces intersymbol interference, which causes the "eye" to close.
- The goal of a channel equalizer is to undo the effect of the channel.
- This will cause the "eye" to open.



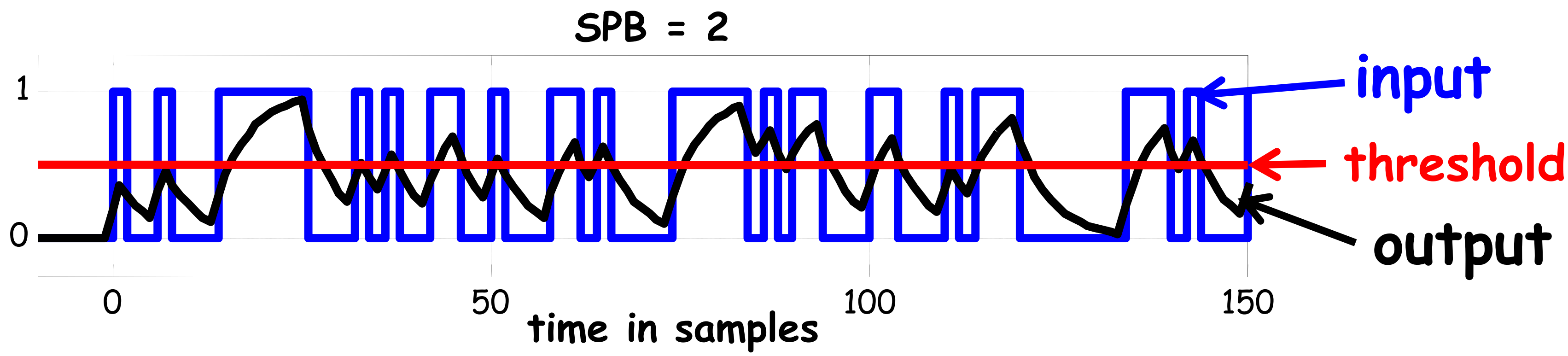
Modeling the Channel

- In order to reverse the effect of the channel, we start with a model of the effect of a channel on the input
- We have seen that the response of the channel to an input $x(n)$ can be described (modeled) in two equivalent ways
  - Model 1:
    - Channel is linear and time invariant
    - Channel has step response $s(n) = k(1-a^{n+1})u(n)$
  - Model 2:
    - If $x(n)$ is the channel input and $y(n)$ is the output,

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$

Intuition for equalizer

- Due to ISI, the output does not always move far enough to cross the threshold in response to a change in the bit.

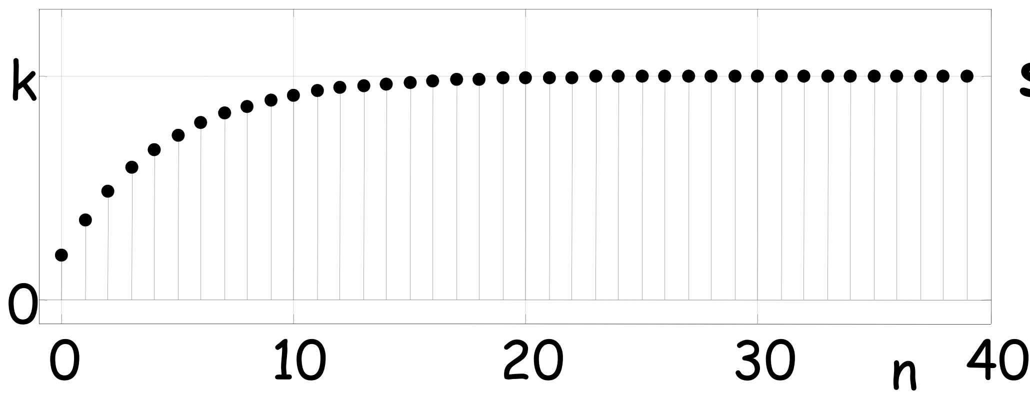


Channel Parameters: c=0, k=1, a=0.8, d=0

- Thus, looking at the value (or level) of the output is not a reliable way to determine the input bit.

Intuition for equalizer

- When the input goes from zero to one,
  - The channel output does not move immediately to k
  - Rather, the output starts to change from zero to k.



$$s(n) = k(1-a^{n+1})u(n)$$

$$0 < a < 1$$

- Can we do better by looking at how the output is changing, rather than the output level?
- How might we combine this information with the output level?

The Equalizer for Model 2

- Inverting $y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$

  we obtain

$$x(n) = \frac{1}{(1-a) \cdot k}\left[y(n) - a \cdot y(n-1)\right]$$

- This is only true if the output of the channel is exactly described by this equation, however, this is not the case due to unmodelled effects, such as nonlinearity and noise or incorrect parameters.
- Thus, this is just an approximation to the input:

$$\tilde{x}(n) = \frac{1}{(1-a) \cdot k}\left[y(n) - a \cdot y(n-1)\right]$$

Interpretation in terms of changes

$$x(n) = \frac{1}{(1-a)\cdot k}\left[y(n)-a\cdot y(n-1)\right]$$

$$= \frac{1}{(1-a)\cdot k}\left[(1-a)y(n)+ay(n)-a\cdot y(n-1)\right]$$

$$= \frac{1}{k}\left[y(n) + \underbrace{\frac{a}{(1-a)}}_{} \underbrace{\left(y(n) - y(n-1)\right)}_{\text{change in output}}\right]$$

current channel output

$$= \begin{cases} 0 & \text{if } a = 0 \\ \infty & \text{if } a = 0 \end{cases}$$

Example

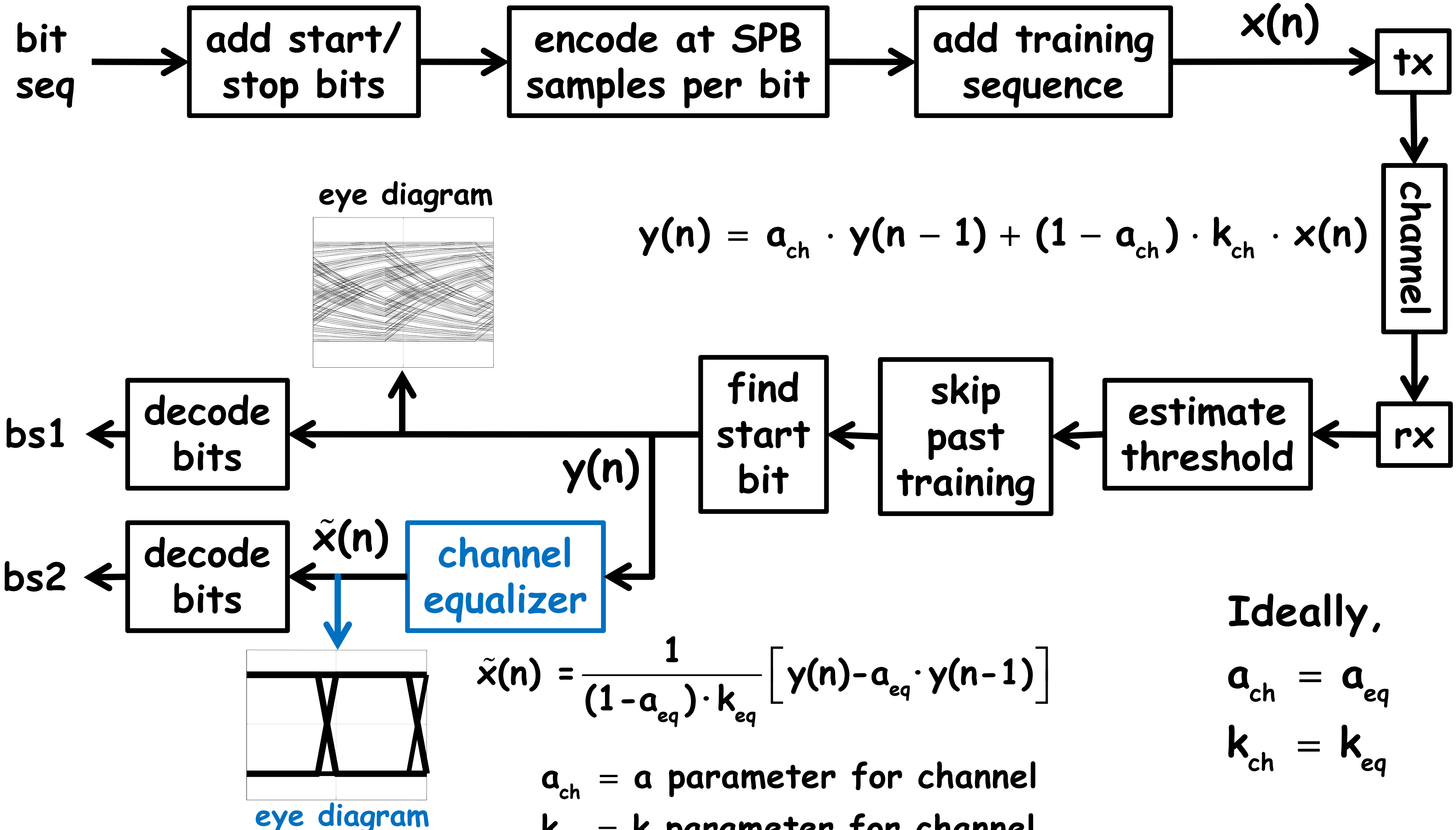
- Suppose we have a channel whose step response can be described as

$$s(n) = \frac{1}{2}\left(1 - \left(\frac{2}{3}\right)^{n+1}\right)u(n)$$

where u(n) is the unit step function.

- What is the equation for the equalizer for this channel?

Adding the Equalizer



$$y(n) = a_{ch} \cdot y(n-1) + (1-a_{ch}) \cdot k_{ch} \cdot x(n)$$

$$\tilde{x}(n) = \frac{1}{(1-a_{eq}) \cdot k_{eq}} \left[y(n) - a_{eq} \cdot y(n-1) \right]$$

a_{ch} = a parameter for channel

k_{ch} = k parameter for channel

a_{eq} = a parameter for equalizer

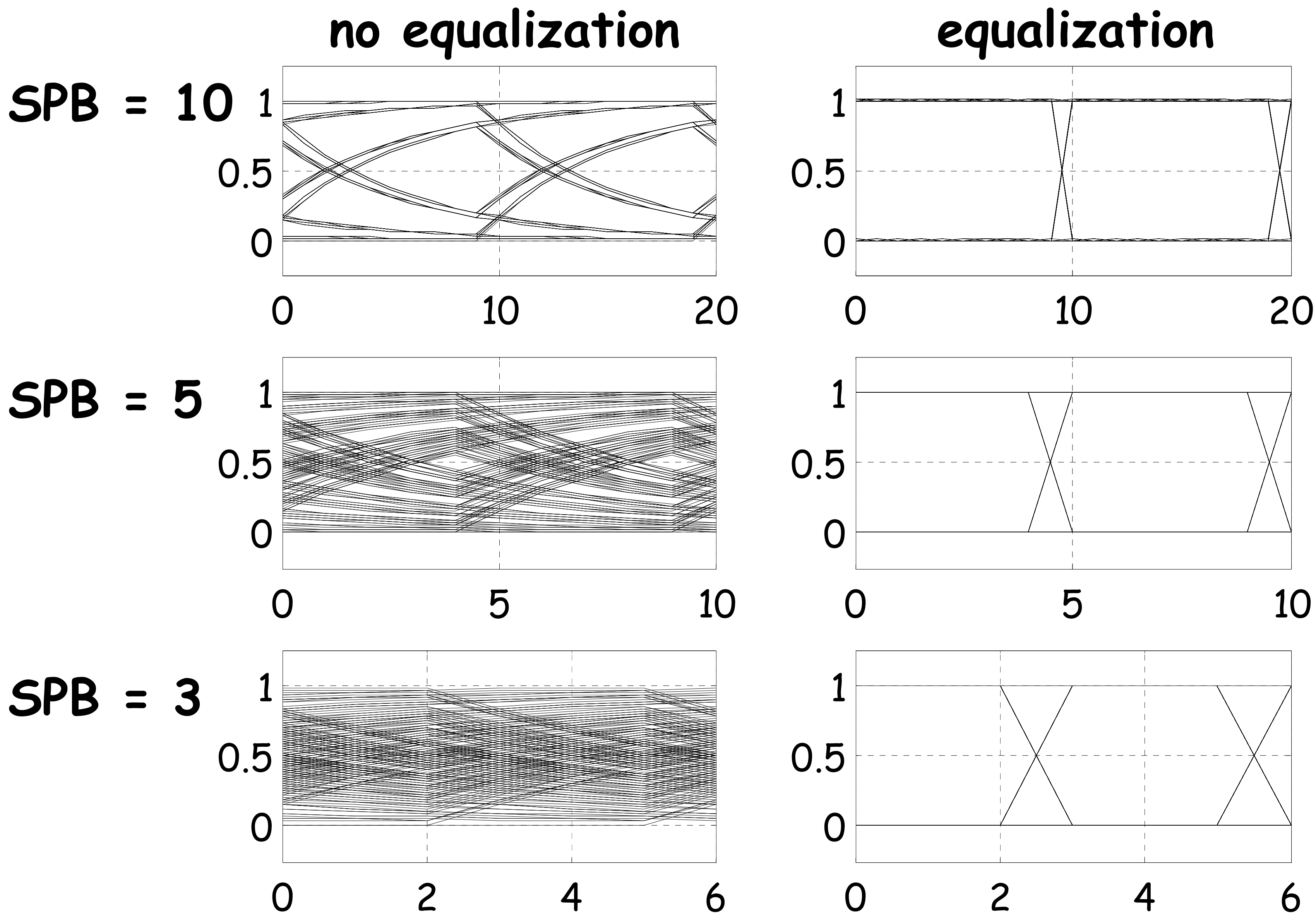
k_{eq} = k parameter for equalizer

Ideally,

$a_{ch} = a_{eq}$

$k_{ch} = k_{eq}$

Effect of Equalization (perfect model)



$c = 0, noise = 0, a_{ch} = a_{eq} = 0.85, k_{ch} = k_{eq} = 1$

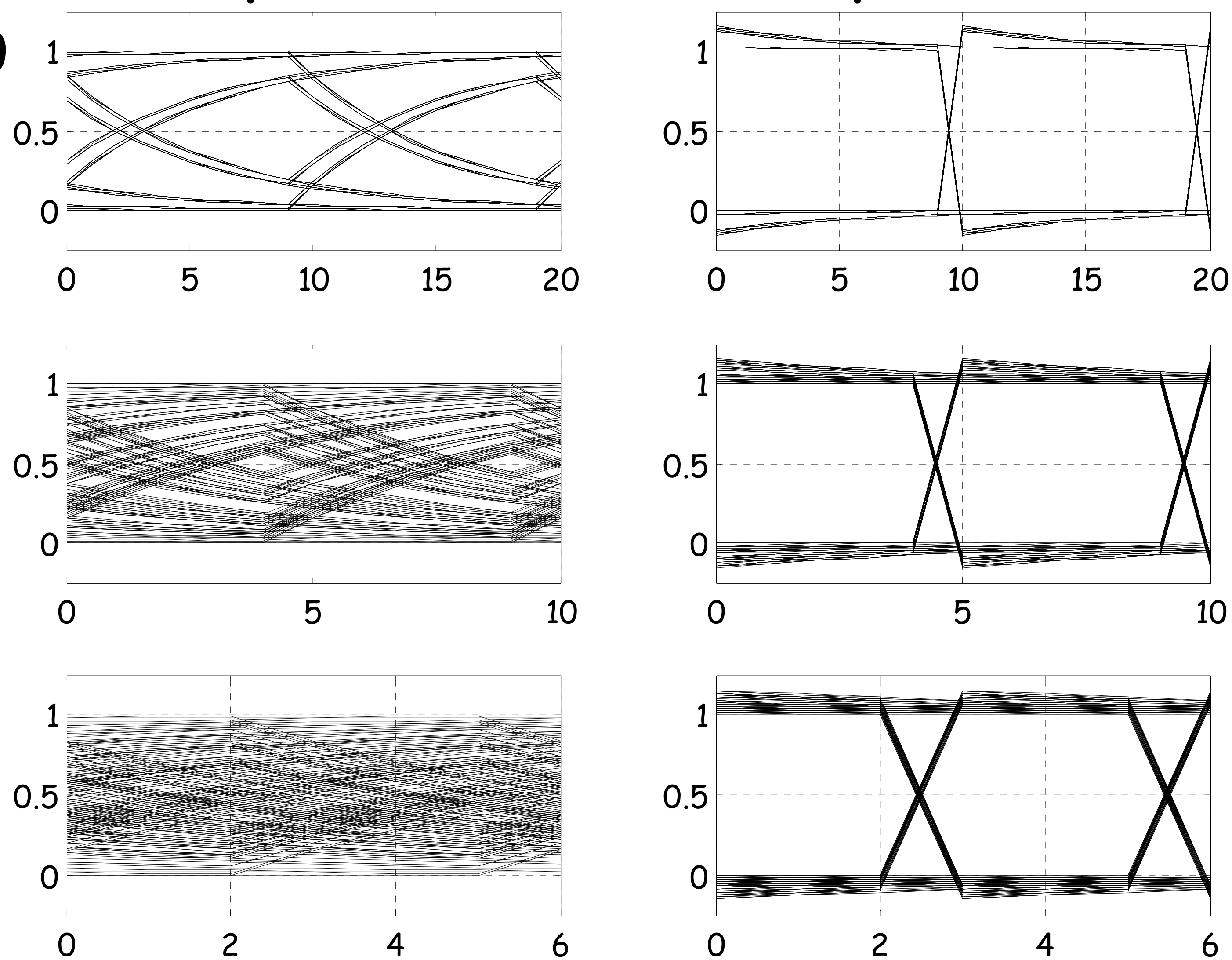
Effect of Equalization (no noise, wrong a)

no equalization | equalization

SPB = 10

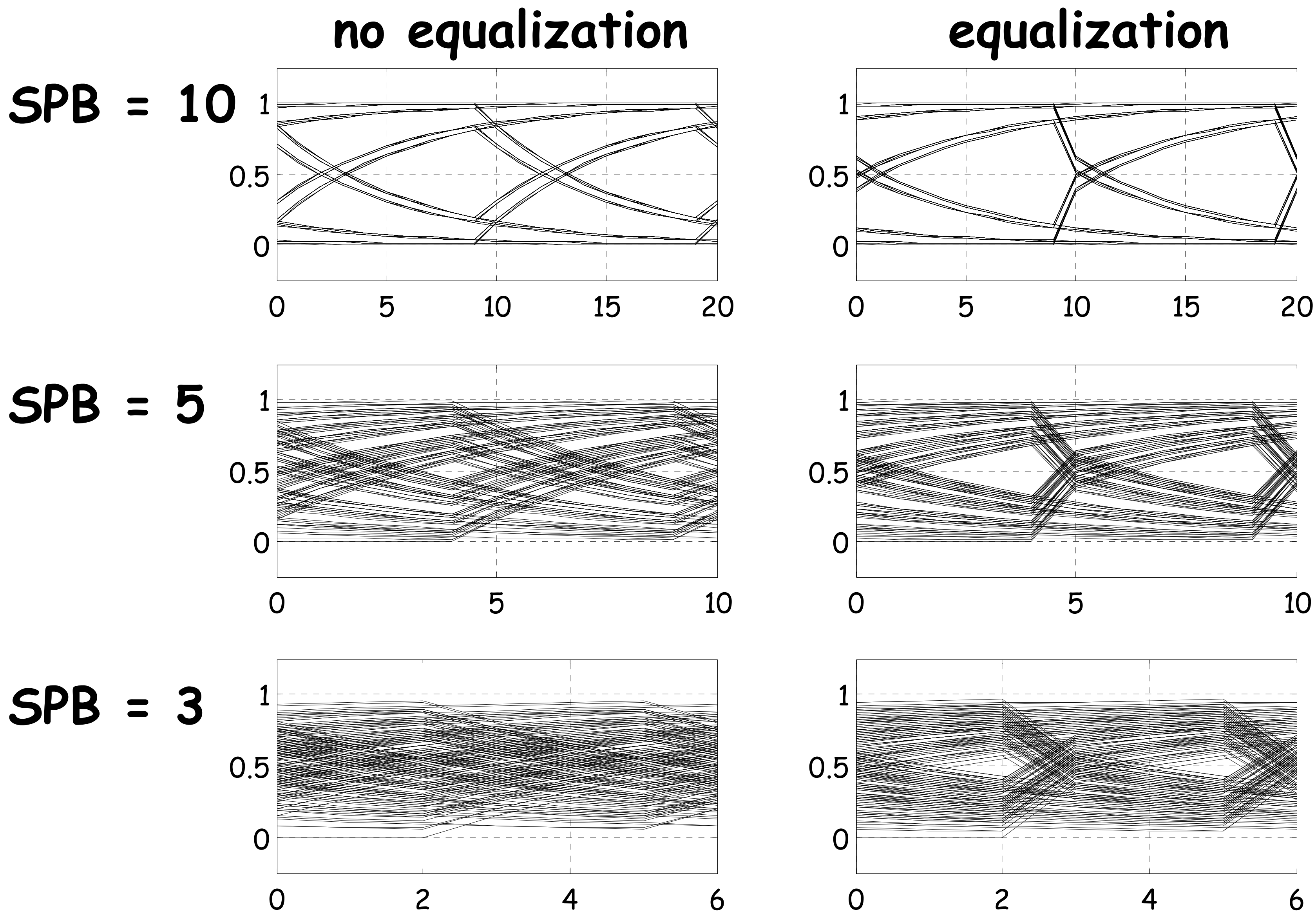
SPB = 5

SPB = 3



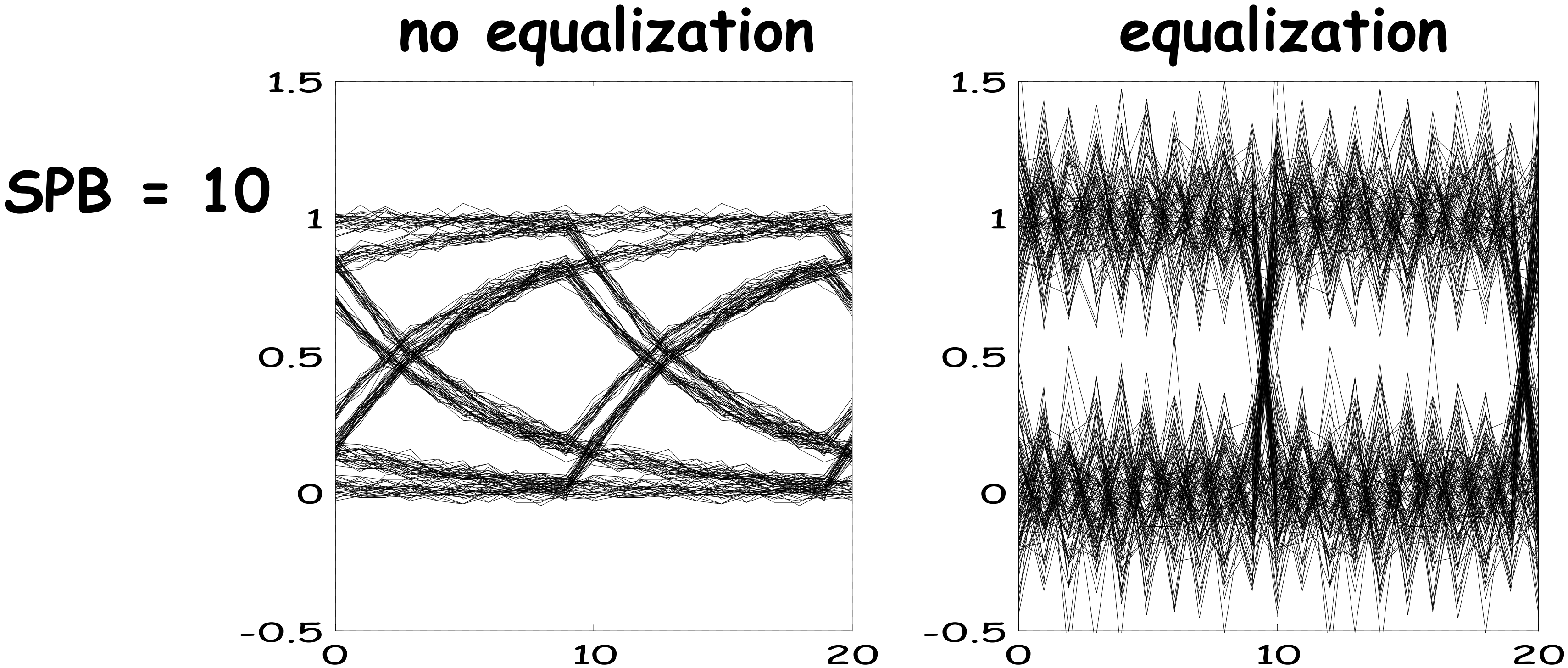
$c = 0, \text{noise} = 0, a_{ch} = 0.85, a_{eq} = 0.87, k_{ch} = k_{eq} = 1$

Effect of Equalization (no noise, wrong a)



$c = 0, \text{noise} = 0, a_{ch} = 0.85, a_{eq} = 0.6, k_{ch} = k_{eq} = 1$

Effect of Equalization (noise, correct a)



$c = 0, \text{noise} \neq 0, a_{ch} = a_{eq} = 0.87, k_{ch} = k_{eq} = 1$

- **Although equalization has "opened" the eye width, it has also increased the size of the noise, "closing" the eye height. Thus, with noise, we need to be careful.**

Summary

- Using a model of the relationship between the channel input and the channel output, we have developed an equalizer that "undoes" the effect of the channel.
- This "opens" the eye, which "closes" due to intersymbol interference.
- The equalizer is <u>robust</u> (still works) even if the parameters of the channel are not correctly estimated.
- However, it may magnify the effect of noise.